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WADD TECHNICAL REPORT 60-200

ANALYSIS OF HEAVY-DUTY PARACHUTE RELIABILITY

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Flight Accessories Laboratory

Contract No. AF 33(616)-6544
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WRIGHT AIR DEVELOPMENT DIVISION
AIR RESEARCH AND DEVELOPMENT COMMAND
UNITED STATES AIR FORCE
WRIGHT-PATTERSON AIR FORCE BASE, OHIO

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FOREWORD

This report was prepared by a task team of the American Power Jet Company, Ridgefield, New Jersey, under the supervision of George Chernowitz, Technical Director. The work was administered by the Advanced Technology Section, Aerodynamic Decelerator Branch, Flight Accessories Laboratory, Directorate of Advanced Systems Technology, Wright Air Development Division, under Contract No. AF 33(616)-6544, "Study, Reliability Determination for Heavy-Duty Parachute Systems." Mr. J. DeWeese was the WADD Project Engineer.

The assistance and cooperation of Mr. DeWeese, Mr. Alfons Hegele, and of other personnel of the Aerodynamic Decelerator Branch are gratefully acknowledged. Thanks are also due for the cooperation of the personnel of the Military and Industrial organizations listed in Appendix D.

ABSTRACT

This report presents a model for the computation of heavy-duty parachute system reliability based on the reliability of the individual components and sub-components of the system and the operational reliability of the system as a whole. The model is applicable to the estimation of system reliability in field use; it can be applied at any desired phase in the development of the system.

Methods of selecting the applicable terms for the model for a specific parachute system, and the details of computation of component reliability values from various types of field use, laboratory test, and engineering data to a pre-selected confidence coefficient are presented. A worked example of a reliability analysis of a hypothetical parachute system is used to illustrate the application of the method.

Numerical results of reliability analyses of parachute packing, reefing line cutter performance, and some solid fabric canopies, and data on parachute materials strength tests usable in the analysis are included. The mathematical derivation of the reliability methodology is presented in an appendix.

PUBLICATION REVIEW

This report has been reviewed and is approved.

FOR THE COMMANDER:

George A. Solt Jr.

GEORGE A. SOLT, JR.
Chief, Aerodynamic Decelerator Branch
Flight Accessories Laboratory

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LIST OF SYMBOLS

E = Expected Value
e = Base of Natural Logarithms
f = Number of Degrees of Freedom
F = Failure Rate
F_p = Number of Parachute Packing Failures
g = Confidence Coefficient
K = Computation Factor Defined in Text
m = Maximum Number
N = Number of Operations or Samples
p = Probability
r = Calculation Factor Defined in Text
R = System Reliability
R_p = Operational Reliability
R_c = Component Reliability
s = Sample Standard Deviation
x = Material Break Strength
$\overline{x}$ = Mean Material Break Strength
X = Calculation Factor Defined in Text
y = Load on Parachute Member
$\overline{y}$ = Mean Load on Parachute Member
z = Parameter Defined as $(\overline{x} - \overline{y})/s$
Z = Number of Suspension Lines
μ = Population Mean
σ = Population Standard Deviation
θ = Calculation Factor Defined in Text

SECTION 1

INTRODUCTION

The study upon which this report is based was undertaken to develop a method for determining the reliability of heavy-duty parachute systems. The method of reliability analysis resulting from this work is intended for use as a tool of the parachute design and development agency. Within the framework established by the project directives, the study is pointed toward the assessment of the reliability of the fully developed parachute system in its field use application.

The first portion of this report deals with the development of a reliability concept applicable to the evaluation of parachute performance. This is followed by discussions of the reliability model developed, means of applying the model to specific parachute systems, and the methods for evaluating the individual terms in the model. The method of application of the model to the analysis of the reliability of a parachute system is illustrated by a worked example, using a hypothetical parachute system. Numerical data on observed failure rates and the results of reliability analyses of specific parachute components, data on parachute materials strength, and a detailed discussion of the mathematical methods developed are presented in three appendixes.

The term "reliability" is used in this report in its commonly accepted sense: it refers to an inverse measure of the expected failure rate, that is, to the figure obtained by subtracting the expected rate of failure of a system from unity. The important concept here is the use of the term "expected" rate of failure". The calculated reliability of a system cannot be used to forecast, on an absolute basis, the performance of a single example of that system in a single use. It gives the "odds", but does not foretell the result of any single event. It refers to the rate of successful uses to be expected when a large number of identical systems are used, or when a given system is used a large number of times, although in the case of the single-use parachute systems considered here, the former statement is the applicable one.

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The rationale used in developing the reliability model in this study assumes that the parachute system is to be evaluated after (1) it has been through its preliminary design stages, (2) a minimum number of models have been built and tested, (3) the results of these tests have been evaluated, and necessary design changes made, (4) modified models have been fabricated and tested, design faults found have been corrected, and revised models tested until the design can be finalized, and (5) a limited number of final models are fabricated and tested to demonstrate the adequacy of the final design.

Because of these assumptions, the reliability model presented is most useful in forecasting the performance of the parachute system after the development process is completed. The type of development process assumed will "shake out" major design errors, both in materials choice and component arrangement. Thus, the factors which contribute to "unreliability" in the final parachute design will be due to "chance" human and material deficiencies, and hence will be susceptible to probability analyses.

The reliability of a parachute system in field use must be relatively high; since the factors which tend to reduce reliability are found and corrected during development, the reliability trend during the development process must be one of continuous increase. Thus, while the application of any reliability model to the determination of system reliability at any intermediate phase in the development process is possible, the value of the results so obtained will be only transitory.

For these reasons, the reliability model developed in this study has been pointed primarily at the determination of the performance of the final product of the parachute system design process. It may be used to forecast this final reliability at any stage in the development. The confidence which may be placed in the accuracy of the reliability calculated, of course, will be greater as the development process progresses and more information on the system is available for the evaluation of the model.

The parachute system reliability model presented in this report is completely general, and can be applied to any type of parachute system. However, in the development of the data necessary to apply it, this study has been pointed specifically toward the evaluation of heavy-duty parachute system reliability. Thus, to use the model to evaluate the reliability of other types of parachute systems, it will be necessary to develop additional data, chiefly on the performance of the system components and the materials used in their fabrication.

In addition, it will be necessary to introduce a "wear out" factor for those systems in which the parachutes may be subjected to repeated use.

SECTION 2

APPLICATION OF RELIABILITY CONCEPT TO PARACHUTE SYSTEMS

In the development of a reliability model for a parachute system, it is necessary to choose, within rather narrow limits, the boundaries defining the system, the use conditions under which its reliability will be considered, and the measures of success in the parachute mission. In the first place, it is necessary to know the exact points in the rather complex sequence of parachute system operation at which the reliability computations begin to apply and that point at which they cease to apply. Further, if the reliability statement computed is to be meaningful, it is necessary to specify the limits of applicability of the system with respect to the physical conditions of use: deployment speed and altitude, load, aircraft maneuvers, etc. Finally, a definition of success or failure of the parachute mission is necessary as a yardstick upon which to base the computation of system reliability.

2.1 Limits of Applicability

In the application of the generalized reliability model developed in this study to the specific problem of the heavy-duty parachute system, the parachute system performance has been considered from the start of its deployment to the instant of touchdown (or upon reaching its equilibrium velocity). Thus, the operation of the device which releases the parachute from the aircraft, and the operation of the ground disconnect device, if any, are not considered in the study of the reliability of the parachute system.

Further, in those cases in which the parachute is deployed automatically from a compartment in the load, the study does not consider the design of the parachute compartment, nor of the explosive-operated or other device which effects the opening of this compartment so that parachute deployment may be initiated.

The above are the system boundaries specified for this study by project directives. If desired, the reliability model can readily be expanded to include other aspects of the parachute system use, including the pre-release operations in the aircraft and the process of touchdown and canopy disconnect where applicable.

In assessing the reliability of any system, it is necessary to define the conditions of system use under which the reliability is to be computed. Thus, in the consideration of a parachute system, maximum load, maximum deployment speed, minimum (and possibly maximum) altitude, allowable aircraft maneuvers, and other factors, under certain conditions, must be defined in order to provide a basis for the computations. If the parachute system is used under less severe conditions, its reliability may, and probably will, be higher than when in use under maximum allowable stress. If it is used under more severe conditions, the reliability will probably decrease as the severity of the use increases, to a point at which the system fails so frequently as to be unusable.

Thus, any reliability figures which may be developed as a result of the work described herein should be regarded as an indication of the performance of a system when it is used under its design conditions. If the conditions of use are less severe than those upon which the calculations are based, the figures may be regarded as minimum reliability. In the case of system use beyond the design limits, the reliability computations based on design limits will not apply.

One additional set of factors which affect the reliability of parachute systems, to some extent at least, are the atmospheric conditions under which they are used. Rain, snow, gusts (both horizontal and vertical) and other meteorological phenomena undoubtedly have some effect on the success of parachute missions during which they occur. However, if such factors were included in a reliability model designed for routine use in forecasting parachute system performance on a world-wide basis, they would complicate the computational process to the point at which the utility of the method would become open to question. Consequently, consideration of the effects of weather conditions upon parachute reliability has been omitted in this study, although the general model developed can be used to calculate their effect if adequate data and computational effort can be made available.

2.2 Measures of Success

In order to apply the concept of reliability to the use of a specific parachute mission, it is necessary to have a firm definition of success, and hence of "failure", of the mission. Obviously, for parachute use in general, the measure of success is the safe delivery of the load to the desired point. This, however, is not an adequate definition for the purposes of this study; two questions arise: (1) What is "safe delivery" in the use of a heavy-duty parachute system in delivering a weapon? and (2) How close to the aim point must the load touch down?

For the usual type of cargo parachute use, these questions are related. Safe delivery depends on the rate of descent being within certain limits, and the trajectory of the system depends on the rate of descent. In the case of the heavy-duty parachute system, the problem of safe delivery is really one of meeting trajectory requirements. Since the type of load delivered may not require pin-point accuracy, there is usually more or less of a range of descent rates which will result in adequate system performance.

In order to avoid the complexities of calculating the effects of various possible types of events during the deployment of the parachute system on the descent rate and landing point, a somewhat different approach to the definition of a successful heavy-duty parachute system mission has been taken herein. For the purposes of evaluation of the reliability model, a parachute failure is defined as the failure of any portion of the parachute construction which will cause an unsuccessful drop, or as a use in which the parachute packing process was improperly performed, so that the parachute deploys in such a manner as to cause failure of the mission.

There are two requirements for the use of this type of criterion of mission success: first, it is necessary to evaluate human performance in parachute packing, and second, it is necessary for the agency doing the evaluation to establish its own criteria of what constitutes failure of the specific parachute system under evaluation.

The first of these problems has been met by a statistical study of the parachute packing failure rate for various types of systems. The second involves no major complications. The evaluation of parachute system reliability is basically an engineering problem, and requires that the engineer understand the operation and construction of the system. Thus, he is in the best position to determine which portions of the construction are critical in the success of the system, and to choose his failure criteria accordingly. Failure criteria chosen in this manner are tailored for the specific parachute system under evaluation; they will undoubtedly produce better results than a rigid set of failure criteria set up for application to all possible parachute systems.

2.3 Summary

The reliability model developed in this study is applicable to the evaluation of the overall expected rate of successful use of a heavy-duty parachute system, within the framework of the definition of success established by the evaluating agency, under the following conditions:

1. Only the parachute system itself is evaluated (deployment initiation devices and ground disconnects are excluded).

2. The parachute system is used within its design limits.

3. The parachute system is a single-use system.

SECTION 3

DEVELOPMENT OF THE RELIABILITY MODEL

There are a number of approaches which may be taken to the determination of the reliability of various types of systems; a considerable volume of material has been published on the subject.[1,2] However, most of the work in the field of reliability has been pointed specifically toward the problems of electronic circuitry, or similarly complex systems, in which the mode of failure is primarily time-based. Thus, the reliability of an electronic circuit is principally a measure of the probability of its successful operation for a given period of time.

Heavy-duty parachute systems, on the other hand, are only required to operate once, and from a practical viewpoint, are likely to fail (within the framework of the definition of failure outlined in the previous section) only during the deployment and opening process. Thus, time-based reliability models, based on testing equipment to failure or to a specific number of cycles of usage (the life test approach[3]) do not generally apply to the present problem.

There are other means of determining the reliability of systems which may be applicable to heavy-duty parachute systems. In general, these are based on trials of a number of systems (or the components of systems), rather than repeated or continuous trials of the same system or small groups of systems. Some of these methods are discussed below.

3.1 Overall Reliability Approach

The least complex approach to the study of the reliability of a single-use system consists of testing a number of systems to determine the failure rate of the sample, and projecting this rate, with allowance for an adequate confidence coefficient, as the failure rate of all systems made to the same specifications. In the application of this approach to heavy-duty parachute systems there are two possible results to each test: pass or fail. Thus, the failure rate is binomially distributed, and the reliability of the system (failure rate subtracted from unity) can be calculated readily from the sample test results.[4]

In presenting the failure rate (or the reliability) calculated in this manner, good statistical practice requires that two figures be given, the computed expected failure rate and the confidence coefficient of this rate, for example: "An expected failure rate of 0.001 (or a reliability of 0.999) at a 90% confidence coefficient." This statement may be interpreted in the following manner: If a large number of sets of samples are tested, it will be found that the reliability of the systems in the sets will be 0.999 at least 90% of the time.

The reasoning behind the formulation of the reliability or failure rate statement in this manner is based on the fact that the statement made is applied to all the systems constructed (or to be constructed) to the given specification, while only a portion of the total number manufactured (or to be manufactured) were actually used in the test. Thus, the reservation about the calculated reliability stated in the confidence coefficient allows for the fact that, by accident, the proportion of failures in the sample tested may not be completely representative of the proportion of failures which will occur when all the systems manufactured are used.

In calculating system reliability by this method from a given series of test results, the confidence coefficient used must be selected by the evaluating agency. Any desired confidence coefficient may be used in the calculation, although in practice the choice of a 100% level will not give meaningful results for reasons discussed below. In the choice of a confidence coefficient for the calculations, it must be realized that the higher the confidence coefficient used, the lower the reliability (or the higher the failure rate), and vice versa. This is illustrated in Figure 1, which presents two sets of curves for the same series of tests, one for a 90% confidence coefficient and the other for 99% confidence. It can be seen that the reliability calculated at the lower confidence coefficient will be greater than that calculated at the higher coefficient, although the two values tend to become closer with a large number of tests.

The choice of a confidence coefficient for reliability calculations in practical cases tends to be dictated by the amount of test data available for study. Unless the test data can be obtained from trials of the system made for purposes other than reliability testing, the cost of doing the testing is probably the controlling factor in the choice of confidence coefficient. Figure 2 is a generalized plot of the relative cost of reliability testing versus the confidence coefficient. It can be seen that the major increase in cost begins at about the 90% confidence coefficient, and that by working at this level, the evaluating agency gets the greatest

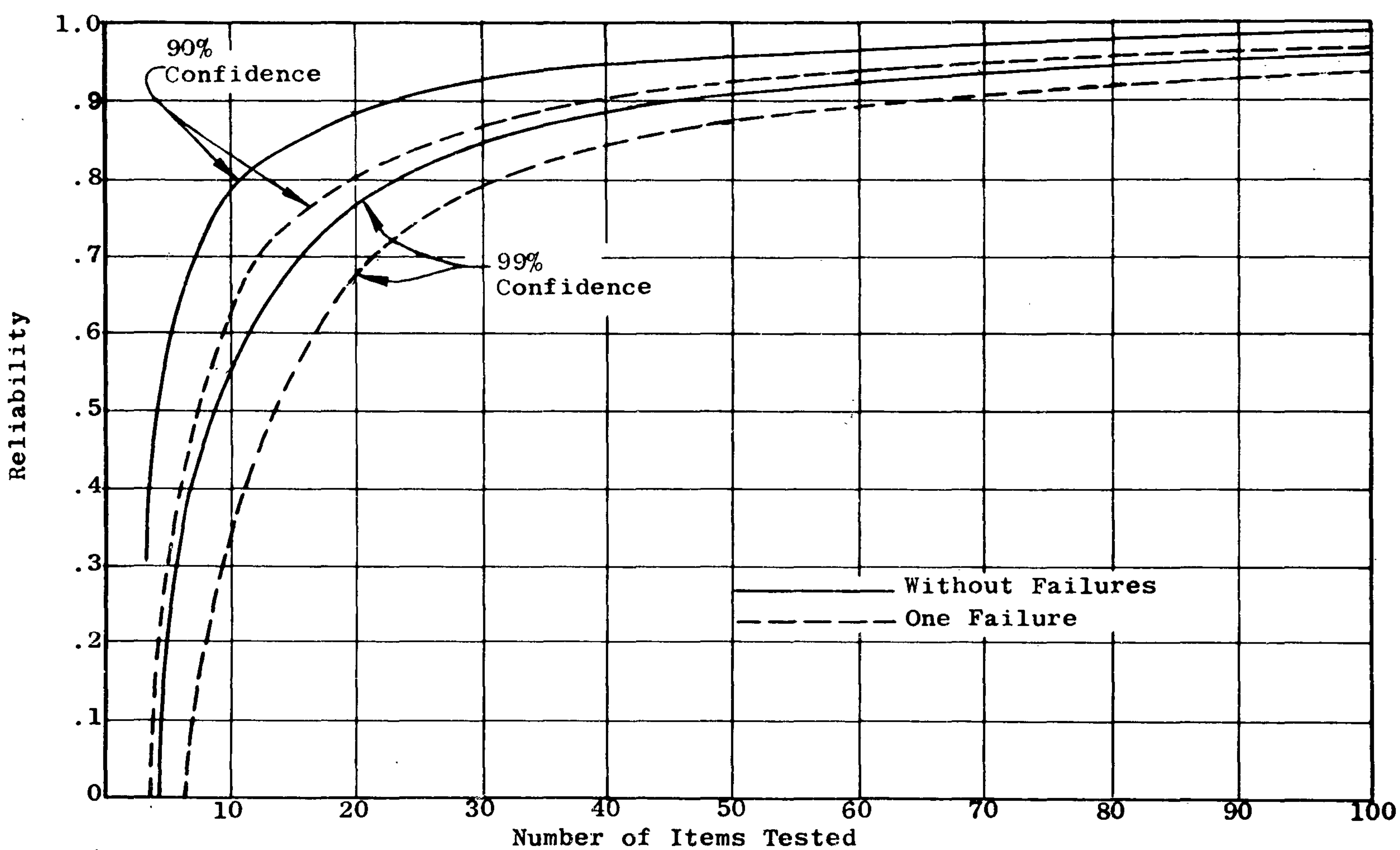

Figure 1. Reliability Levels for a Series of Tests With and Without Failures.

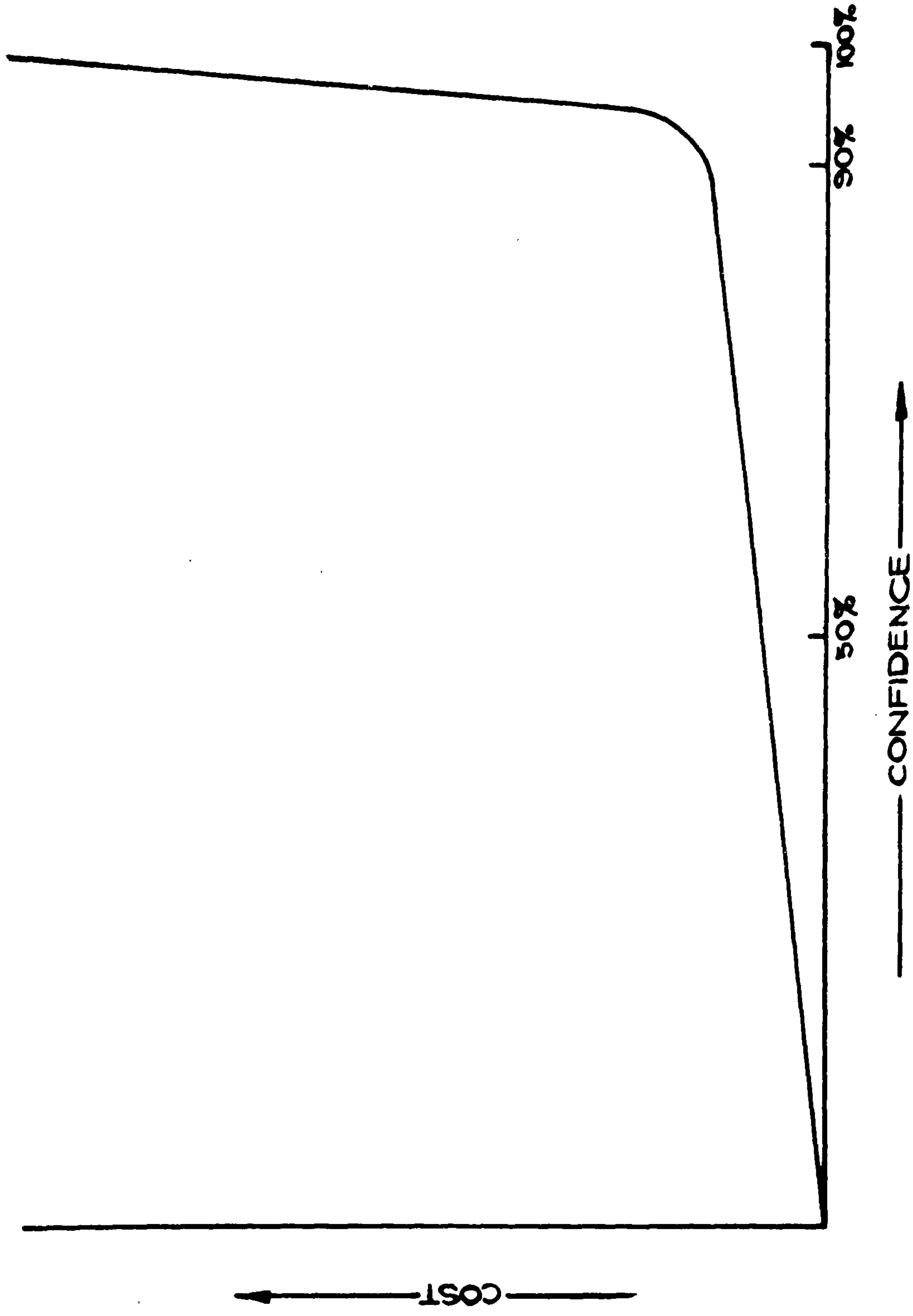

Figure 2. Cost of Testing to a Given Confidence Coefficient.[5]

return for the amount of money spent. Obviously, 100% confidence has no real meaning in this concept, since its achievement requires an infinite number of tests. That is, to achieve 100% confidence in a reliability statement requires knowledge of the results of every use of the system, including advance knowledge of future uses.

When the problem of determining the reliability of heavy-duty parachute systems is considered, from a confidence coefficient viewpoint, the information presented in Figure 1 becomes the key issue. Even if no failures whatsoever are encountered in testing, over twenty systems must be tested to be sure of 0.9 reliability at a 90% confidence coefficient. If assurance of 0.98 reliability is desired at a 90% confidence coefficient, about 100 systems must be tested without failure. If greater confidence in the results is desired, e.g. 99%, about 50 systems must be tested without failures to assure 0.9 reliability; the testing of 100 systems will assure only about 0.93 reliability.

If the reliability model is to be applied to a system under development, to forecast its performance when it reaches the field-use stage, it is doubtful whether adequate use data would be available for such computations. The data needed are the results of tests of the system in its final configuration. Thus, data on development test series during which the system configuration is changed to eliminate defects are not applicable, both because of difference in the physical make-up of the parachute system and because of possible differences in the use conditions.

Use data on the final configuration of the system are rarely available in any quantity, even at the end of the development phase. Indeed, when the applications of the heavy-duty parachute system are considered, it can be seen that the generation of sufficient use data for the type of reliability computation described above is very unlikely, even after the system has been developed for field use.

For any item as complex and as expensive to test as a heavy-duty parachute system, the number of tests required for reliability computations by the method described above is obviously excessive. By the use of sequential analysis,[6,7] the number of tests which must be performed for a given confidence coefficient can be reduced to some extent (under certain conditions by as much as 50%[8,9]), but the amount of testing required for this type of analysis is still quite expensive.

The applicability of this type of analysis to the heavy-duty parachute reliability problem comes in the study of individual component reliability (see below), when certain components of the heavy-duty system (e.g., reefing line cutters, solid canopies) for which extensive use data are available are also used in other parachute systems.

3.2 Component Reliability Approach

The overall approach to system reliability discussed above considers the parachute system as a unit, and measures its reliability by measuring the performance of the whole system. The component approach to system reliability, on the other hand, measures the reliability of each component of the system, and computes the reliability of the complete system from the reliability of its components and the operational reliability of the combination of components. The advantage of the component approach is that it can utilize laboratory test data, engineering computations, or actual field use experience with identical components in other systems to compute the reliability of the complete system, without requiring extensive testing of the entire system.

3.2.1 Basic Concept

The basic mathematical model describing the component reliability method is

$$R = R_p . R_{c_1} \ldots R_{c_n} \tag{1}$$

where

R = System reliability

R_p = Operational reliability

$R_{c_1} \ldots R_{c_n}$ = Reliability of individual components.

In this model, the term "reliability" has its usual meaning, $R = (1-F)$, where F represents the failure rate.

The operational reliability term in this model represents the rate of failure which can be expected because parachute system components must function in a specific manner and in a specific sequence during deployment and inflation of each of the individual canopies in the system. The component terms, on the other hand, represent the rate of failure which can be expected to result from inherent weaknesses of each of the components of the system. Together, these have been taken as the total significant causes of failure in a parachute system.

3.2.2 Data Required

To use the above model in a practical evaluation of parachute reliability, it is necessary to develop methods of evaluating the specific terms in the model.

Actually, there are two types of terms to be evaluated, those representing the reliability of the individual parts of the system (the number of terms required, of course, will depend on the characteristics of the system being studied -- see next section) and the term which represents the reliability of the operation of these parts as a system.

3.2.2.1 Component Terms. In evaluating the component reliability terms, three possible approaches to the calculation of the probability of failure of a given component have been studied. The preferable method of calculation utilizes test data on the component generated in its use in other parachute systems, or by deliberate testing, laboratory or field, to provide a basis for calculation. The second type of calculation utilizes data on the stresses on various components of the parachute system derived from instrumented drop tests and data on the strength of the materials from which the component is constructed to compute the probability of component failure. The third, and least preferable, approach utilizes engineering computations on the expected stresses on the various portions of the component, together with the data on the strength of the component materials, to compute the potential failure rate.

3.2.2.2 Operational Term. In developing a method for evaluating the operational term, the parachute deployment process was studied, considering the fact that the reliability computations represent the performance of the parachute system after a field test and redesign process which should correct any avoidable operational difficulties. The results of this work indicate that what are here called operational failures, in field use, will be caused by human error in the parachute packing process, since the parachute development process will result in a product which will function properly if (1) it is used under its design conditions, (2) no materiel failures occur, and (3) the parachute is packed properly. The basic assumptions under which the reliability of the system is studied assume that condition (1) will be true. Failures due to materials (component failures) are considered separately. Thus, from a practical viewpoint, the operational failure term can be evaluated, based on the rate of human error in the parachute packing process.

The details of the calculation of component reliability by each of the three methods, and the computation of parachute packing (operational) reliability are covered in the next section of this report.

SECTION 4

EVALUATION OF COMPONENT RELIABILITY MODEL

The problem of computing "exact" values of the reliability of a heavy-duty parachute system with any type of a model is a complex one; in general, to obtain "best" estimates of reliability requires extremely heavy computation. However, mathematically sound techniques of approximation exist, which, if properly applied to the component reliability model, will yield quite satisfactory estimates of reliability with a significant easing of the computational load. The latter approach is the one followed in the subsequent discussion.

4.1 Selection of Component Terms

As a first step in the analysis of parachute system reliability with the component model, the complete system must be subdivided into simpler independent component systems, each of which is subject to reliability analysis as a unit. The estimates of component reliabilities thus established are then combined to give the overall reliability. Of course, a considerable amount of judgment must be applied in the subdivision of the parachute system into components; the minimum number of components for which failure rates can be computed is the most desirable result. The components chosen should be independent entities with respect to their possible modes of failures in the use of the system.

Component reliabilities will vary over a broad range; obviously those components with very high reliability will not affect the overall reliability of the system to any significant degree if components of lower reliability are also present. (Since the numerical values of reliability may range from zero to unit, it is obvious that the lowest component reliabilities will have the greatest effect on the overall system reliability. For example, if the system is composed of three components with reliabilities of .999, .999, and .900 respectively, the overall reliability -- .999 x .999 x .900 -- will be .898, which differs by only two tenths of one percent from the value for the lowest component, .900.)

It is obvious that a parachute system will have a relatively large number of components, some of which, such as the canopy, will have a large number of subcomponents. Any of these components or subcomponents may cause failure of the

system if the component or subcomponent fails. If all components and subcomponents are analyzed for reliability, a very large amount of effort is required. Much of this may be eliminated, however, by a preliminary analysis to determine the components and subcomponents most likely to fail under normal load. These, of course, will have the lowest reliability, while the components which are least likely to fail will have the highest reliability.

Thus, in the early stages of the analytical effort, the classification of all components of the system into two major groups, highly reliable and less reliable, can facilitate the computation of the overall reliability by eliminating many terms which do not contribute materially to the magnitude or accuracy of the final result.

It is not possible to write hard-and-fast rules for the classification of parachute system components in this process. The experience and judgment of the engineer evaluating the system is the key factor in making such decisions. In general failures which will affect mission success of hardware items, deployment bags, reefing lines, break cords, radial canopy reinforcements, and some other components (in field-use, properly packed parachute systems used under their design conditions), appear to be so rare as to be generally negligible, although exceptions may occur in parachute systems of unusual design. On the other hand, consideration of the failure rates of such components as risers, bridles, suspension lines, reefing line cutters, mechanical disconnect systems, etc. will probably be required with much greater frequency.

The optimum procedure recommended, following the definition of the components of the system, is a preliminary qualitative analysis of the system to select those components known to be very highly reliable and those known to be of marginal reliability. The middle-range components can then be subjected to a rough quantitative analysis to eliminate other highly reliable items from consideration. The remainder of the components, and those deemed marginal by solely qualitative considerations, make up the group for which component reliabilities must be computed.

4.2 Computation of Operational Reliability

To illustrate the concept of reliability and its computation, the specific concept of operational reliability, that is, of the parachute packing reliability, which will probably be one of the more important factors in the overall result, will be considered here in detail. Suppose that on the basis of past experience with parachutes similar to the one under

study, there were F_p packing failures in N observed parachute packings. On the basis of these empirical data, an estimate of the reliability of packing this type of parachute, to be denoted by R_p, is desired. The maximum likelihood estimate of R_p is given by

$$\frac{N - F_p}{N} = 1 - \frac{F_p}{N} \qquad (2)$$

However, it is quite possible that the "true" reliability (that is, the reliability for a very large number of packings) is either less or greater than this maximum likelihood estimate. To handle this situation, a more refined measure of reliability is needed. Roughly speaking, if an estimate of the reliability of a system is made, there is associated with that estimate a probability of the estimate being incorrect. The lower the estimate of the reliability (expressed as a probability of "at least" a given fraction of successes), the higher is the probability of the estimate being correct. The estimate of the reliability will be denoted in the present case by $R_{p,g}$; the probability of the estimate being correct will be called the confidence coefficient (denoted by g). A probabilistic interpretation of these concepts is that if in many empirical studies with F_p failures out of a total of N cases, the reliability is estimated to be at least $R_{p,g}$, then the estimate will be correct on the average of at least g (percent) of the time.*/

In order to compute $R_{p,g}$, it must be recalled that if the "true" reliability is R, then in a single packing the probability of failure is (1-R) and the probability of success is R. Using the binomial distribution, the probability of F_p or less failures in N packings is given by

$$\sum_{i=o}^{F_p} \frac{N!}{i!\ (N-i)!} (1-R)^i\ R^{N-i} \qquad (3)$$

---

*/ In the literature, the concept is usually stated in terms of the statement being correct on the average of exactly g (percent) of the time; the modification here presented is for purposes of flexibility in using approximations.

As R decreases, the expression in formula (3) also decreases. The estimate $R_{p,g}$ will be that value of R which causes formula (3) to be equal to 1-g, for then the probability of obtaining more than F_p failures in N tests will be g. If F_p is small while N is large (that is, if there are not many packing failures in a large number of parachute uses), the Poisson approximation to the binomial distribution may be used. This may all be summarized by the equation

$$1-g = \sum_{i=o}^{F_p} \frac{N!}{i!\ (N-i)!} (1-R)^i R^{N-i} \tag{4a}$$

$$\approx \sum_{i=o}^{F_p} \frac{N^i\ (1-R)^i\ e^{-N(1-R)}}{i!} \tag{4b}$$

When $R_{p,g}$ is computed from equation (4), it is then possible to state that the reliability lies between $R_{p,g}$ and 1 with the assurance of being correct given by the confidence coefficient g.

To facilitate computation of R to a given confidence coefficient, Table I may be used. The value in the table for the observed number of failures in the column headed by the desired confidence coefficient is divided by the total number of trials to obtain the expected failure rate. Subtracting this rate from unity gives the desired reliability at the confidence coefficient used.[*]

Further discussion of parachute packing reliability and the results of the analysis of available parachute packing failure data by these methods are presented in Appendix A.

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*/ It should be noted that the values given in Table I are based on the approximate form, Equation 4b. Thus, it is applicable to cases in which ten or more items are tested, and in which the ratio of failures to trials is no greater than 0.2 (not more than 2 failures in 10 trials). Beyond this limit the exact form, Equation 4a, should be used. Tables are available to facilitate such computations.[21,22]

Table I. Table of Values for Computing Reliability to a Selected Confidence Coefficient From a Series of Trials[10]

No. of Failures	Confidence Coefficient 90%	94%	95%	96%	96.5%	97.5%	98%	98.7%	99%
0	2.28	2.94	3.00	3.22	3.37	3.68	3.87	4.36	4.58
1	3.89	4.68	4.74	5.02	5.19	5.56	5.79	6.36	6.61
2	5.32	6.22	6.30	6.61	6.80	7.21	7.46	8.10	8.38
3	6.68	7.67	7.75	8.09	8.30	8.75	9.03	9.71	10.0
4	7.99	9.07	9.15	9.52	9.74	10.2	10.5	11.2	11.6
5	9.28	10.4	10.5	10.9	11.1	11.6	12.0	12.8	13.1
6	10.5	11.8	11.8	12.2	12.5	13.0	13.4	14.2	14.5
7	11.8	13.0	13.2	13.6	13.8	14.4	14.7	15.7	16.0
8	13.0	14.3	14.4	14.9	15.2	15.8	16.1	17.0	17.4
9	14.2	15.6	15.8	16.2	16.5	17.1	17.5	18.4	18.8
10	15.4	16.9	17.0	17.5	17.8	18.4	18.8	19.7	20.1
11	16.6	18.1	18.2	18.8	19.0	19.7	20.1	21.0	21.5
12	17.8	19.4	19.5	20.0	20.3	21.0	21.4	22.4	22.8
13	19.0	20.6	20.7	21.2	21.6	22.2	22.7	23.7	24.1
14	20.2	21.8	21.9	22.5	22.8	23.5	23.9	25.0	25.5
15	21.3	23.0	23.1	23.7	24.0	24.8	25.2	26.3	26.7
16	22.5	24.2	24.3	24.9	25.2	26.0	26.4	27.6	28.0
17	23.6	25.4	25.5	26.1	26.5	27.2	27.7	28.8	29.3
18	24.8	26.6	26.7	27.3	27.7	28.5	28.9	30.0	30.6
19	25.9	27.8	27.9	28.5	28.9	29.7	30.1	31.3	31.8
20	27.0	28.9	29.1	29.7	30.1	30.9	31.4	32.6	33.1

$$1 - \frac{\text{Table Value}}{\text{No. of Trials}} = \text{Reliability at Chosen Confidence Coefficient}$$

4.3 Computation of Component Reliability

The other type of factor in the model from which the overall result is calculated represents the reliability of specific components, to be denoted by R_c. It is safe to assume that R_p and R_c are independent of one another, so that each may be considered by itself. As an example of a major component, the parachute canopy, including the suspension lines may be used. If the canopy under analysis (or any other component of the parachute system) is used in another parachute system for which an adequate body of test or use data are available, the analysis of the component reliability for the desired confidence coefficient chosen, proceeds as described above for packing reliability.

If use or test data are not available for the specific component, an alternate method of analysis is required. Again, taking the canopy as an example, it can be seen that the canopy consists of several sub-components, such as the canopy fabric (or ribbons), the vent band, the skirt band, the suspension lines, etc. Hence, the canopy can be considered as a combination of subsystems in series (in a probability sense). If no data are available from which the reliability of the complete canopy can be calculated, it will be necessary to analyze the reliability of the canopy subcomponents. Some simplification of this task may be effected by preliminary analysis of subcomponents to eliminate consideration of those of very high reliability (see above). To simplify subcomponent analysis further, the subcomponents which must be analyzed may be treated as components of the complete system directly, and be represented by individual R_c terms in the overall system reliability model, Equation (1).

4.3.1 Use of Instrumented Test Results and Materials Strength Data

In the analysis of the reliability of parachute components (or subcomponents) for which an adequate number of tests are not available, the preferred method is the one utilizing the results of instrumented drop tests on actual parachutes which provide load data on the components of interest, together with the results of tensile strength tests on the materials from which the parachute is fabricated.

Unfortunately, in practical cases, the available instrumented test data on the parachute component for use with the materials strength distribution data will usually be based upon very few tests. However, the application of the Non-Central t-Distribution 17/ to the instrumentation and materials strength data will allow the computation of component

reliabilities and their associated confidence coefficients with a degree of precision adequate for the analysis. The types of data required and the methods of computation are discussed below; the mathematical derivations are presented in Appendix C.

The materials of construction used in parachutes are produced in relatively large quantities by machine production methods, using yarns produced in similar (or larger) quantities by other machine methods. The final products (webbings, cords, tapes, cloths, etc.) are inspected and tested, generally more than once, and defective materials rejected. Under such circumstances it might be expected that the various batches of materials produced would be uniform in strength, or nearly so.

In practice, however, materials tests indicate that there is an appreciable batch-to-batch variation in break strength of the various fabric items used in parachute construction. Investigation of the results of tensile strength tests on a number of parachute materials (see Appendix B) indicates that the distribution of strength for all of these materials is essentially normal. That is, if a large number of samples, N, are tested and the results plotted as a graph, a symmetrical bell-shaped curve similar to that in Figure 3 will be obtained. The mean (average) value, $\bar{x}$, of the break strength (x) corresponds to the peak of the curve. The actual shape of the curve is determined by the spread of the values observed, which can be measured by the standard deviation, s_x:

$$s_x = \sqrt{\frac{\sum_{i=1}^{N} (x_i - \bar{x})^2}{N-1}} \qquad (5)$$

Study of the results of instrumented parachute drop tests, and consideration of the processes by which the loads measured are generated, indicate that these values are also distributed approximately normally. Thus, in the following discussion it will be assumed that both the load values and the materials break strength data are distributed normally.

Let

$\bar{x}$ = sample mean of the break strength tests (with allowance for sewing, etc. -- see following sections)

s_x = sample standard deviation of the break strength tests (with corrections for multiple layers -- see following sections)

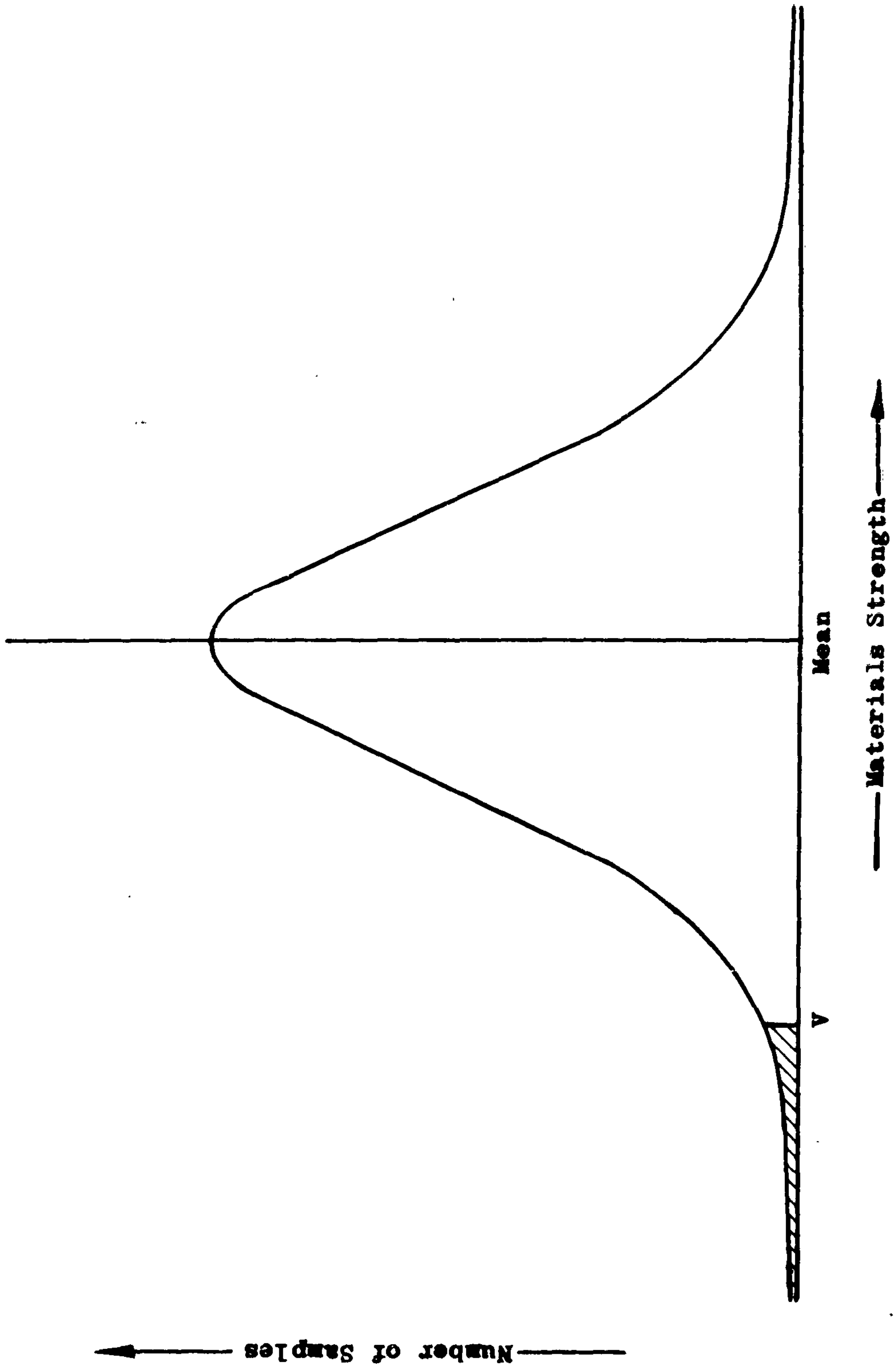

Figure 3. Shape of the Normal Curve.

N_x = number of break strength tests

f_x = number of degrees of freedom of break strength tests = $N_x - 1$

$\bar{y}$ = sample mean of the loads of the instrumented drop tests

s_y = sample standard deviation of the instrumented drop test loads (see Equation 5)

N_y = number of instrumented drop tests

f_y = number of degrees of freedom of instrumented drop tests = $N_y - 1$

The convention is established that the set of data with the smaller standard deviation is considered as s_1, N_1 and f_1 in the following, with the other set designated s_2, N_2 and f_2.

Then, define

$$z = \frac{\bar{x} - \bar{y}}{s} \tag{6}$$

$$s = \sqrt{s_x^2 + s_y^2} \tag{7}$$

$$N = \frac{s_1^2 + s_2^2}{\frac{s_1^2}{N_1} + \frac{s_2^2}{N_2}} = N_2 \frac{1 + r}{1 + \theta r} \tag{8}$$

$$r = \frac{s_1^2}{s_2^2} \tag{9}$$

$$\theta = \frac{N_2}{N_1} \tag{10}$$

$$f = \frac{(s_x^2 + s_y^2)^2}{\frac{s_x^4}{f_x} + \frac{s_y^4}{f_y}} = f_2 \frac{(1 + r)^2}{1 + gr^2} \tag{11}$$

$$g = \frac{f_2}{f_1} \quad (12)$$

$$X = \sqrt{\frac{N}{f}} \cdot z \quad (13)$$

Then, for a given confidence coefficient, g, the use of the graphs of the Non-Central t-Distribution (Figures 4, 5, and 6), or of the tables in Appendix C with the computed values of X and f (Equations 11 and 13) allows the determination of a factor K. Substitution of this value in

$$\mu/\sigma = K \cdot \sqrt{\frac{f+1}{N}} \quad (14)$$

allows calculation of a value of μ/σ with which to enter the table of Normal Distribution, Table II, to find $R_{c,g}$, the desired reliability. Having computed $R_{c,g}$, the statement can be made that the component reliability lies between $R_{c,g}$ and 1 with the assurance of being correct given by g.

The computation of overall reliability to a pre-selected confidence coefficient requires that the component reliability terms be computed to confidence coefficients which will give the desired result in the final calculations (see paragraph 4.4 below). However, tables of the Non-Central t-Distribution are available for the computation of reliabilities to only a limited group of confidence coefficients in the range of interest: 90%, 95% and 99%.

If values intermediate to these are required, it will be necessary to interpolate graphically for the desired confidence coefficient. The simplest method is to compute reliability of the component at each of the three confidence coefficients, plot the reliability values against the confidence coefficients, and interpolate for the reliability at the desired confidence on the graph. An example of this process is given in Section 5.

In applying the above technique to the computation of parachute component reliability, it should be noted that the mean and standard deviation of the instrumented drop test results must be divided among the number of load-bearing members under consideration when a group of subcomponents share the load. Thus, it is necessary to determine the mean load per line and the standard deviation per line ($\bar{x}/Z$ and s_x/Z where Z = number of suspension lines).

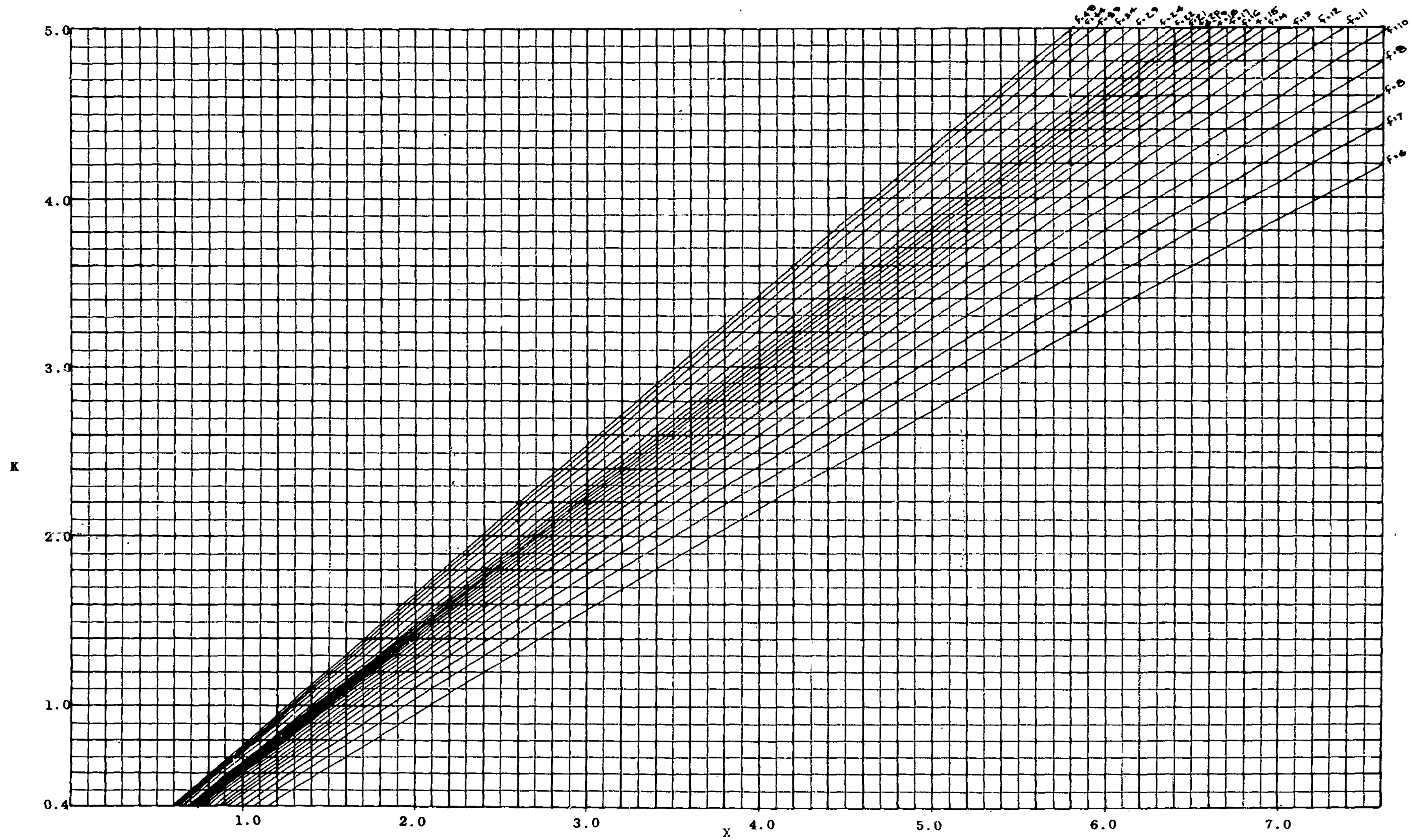

Figure 4. Plot of Percentage Points of the Non-Central t-Distribution, 90% Confidence

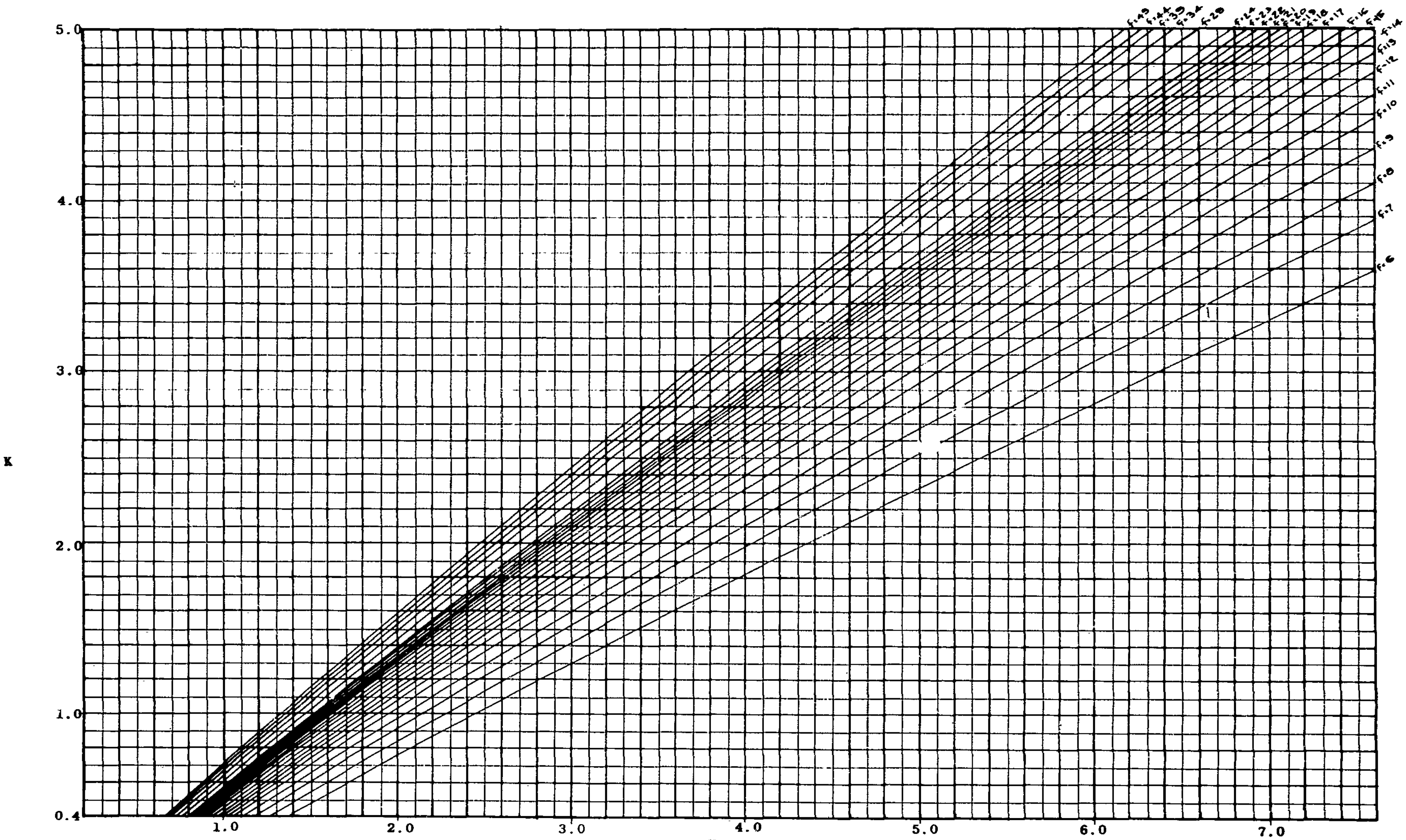

Figure 5. Plot of the Percentage Points of the Non-Central t-Distribution, 95% Confidence

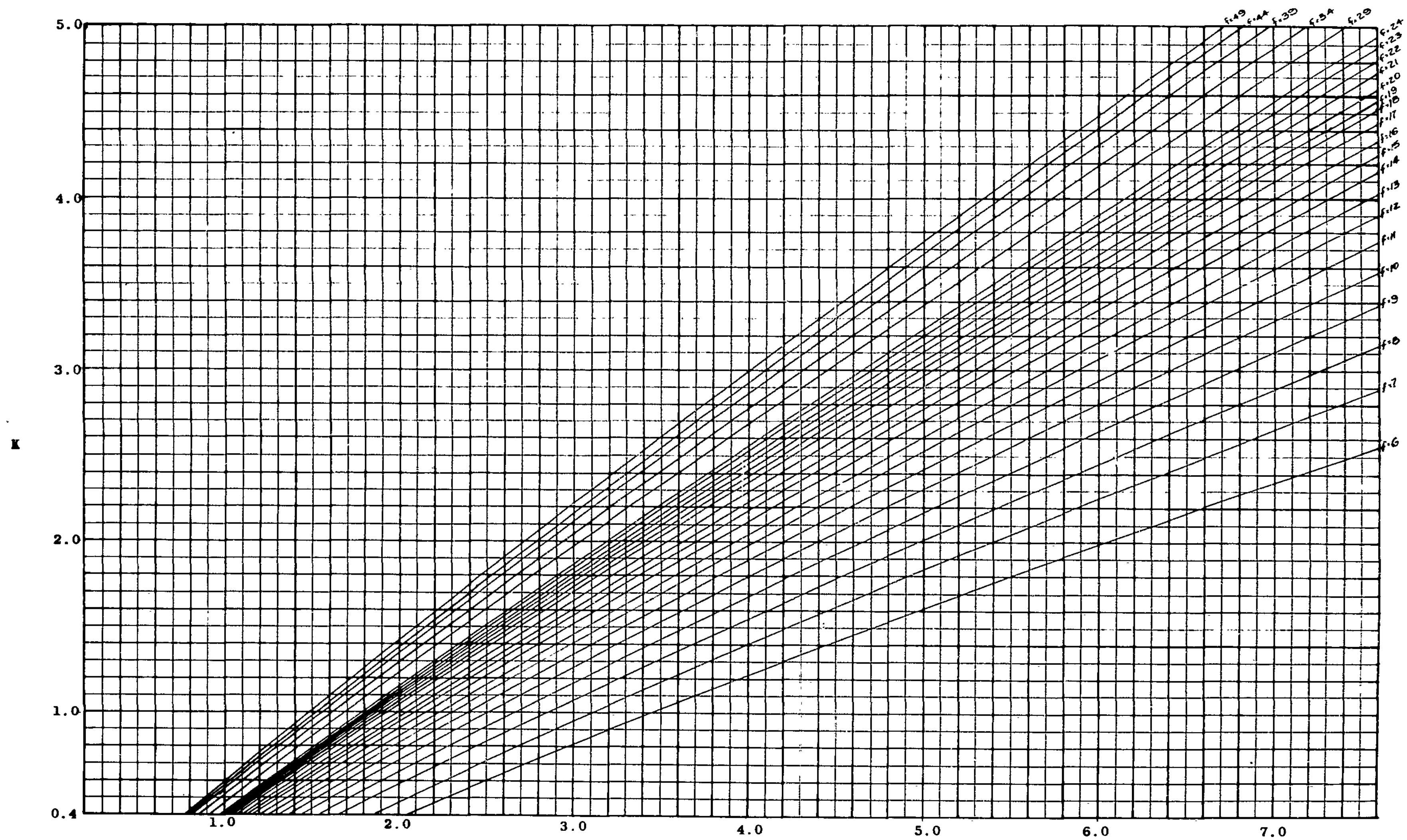

Figure 6. Plot of the Percentage Points of the Non-Central t-Distribution, 99% Confidence

Table II. Table of the Normal Distribution

$\frac{\mu}{\sigma}$	Reliability	$\frac{\mu}{\sigma}$	Reliability	$\frac{\mu}{\sigma}$	Reliability
1.00	.8413	1.35	.9115	1.70	.9554
1.01	.8438	1.36	.9131	1.71	.9564
1.02	.8461	1.37	.9147	1.72	.9573
1.03	.8485	1.38	.9162	1.73	.9582
1.04	.8508	1.39	.9177	1.74	.9591
1.05	.8531	1.40	.9192	1.75	.9599
1.06	.8554	1.41	.9207	1.76	.9608
1.07	.8577	1.42	.9222	1.77	.9616
1.08	.8599	1.43	.9236	1.78	.9625
1.09	.8621	1.44	.9251	1.79	.9633
1.10	.8643	1.45	.9265	1.80	.9641
1.11	.8665	1.46	.9279	1.81	.9649
1.12	.8686	1.47	.9292	1.82	.9656
1.13	.8708	1.48	.9306	1.83	.9664
1.14	.8729	1.49	.9319	1.84	.9671
1.15	.8749	1.50	.9332	1.85	.9678
1.16	.8770	1.51	.9345	1.86	.9686
1.17	.8790	1.52	.9357	1.87	.9693
1.18	.8810	1.53	.9370	1.88	.9700
1.19	.8830	1.54	.9382	1.89	.9706
1.20	.8849	1.55	.9394	1.90	.9713
1.21	.8869	1.56	.9406	1.91	.9719
1.22	.8888	1.57	.9418	1.92	.9726
1.23	.8907	1.58	.9430	1.93	.9732
1.24	.8925	1.59	.9441	1.94	.9738
1.25	.8944	1.60	.9452	1.95	.9744
1.26	.8962	1.61	.9463	1.96	.9750
1.27	.8980	1.62	.9474	1.97	.9756
1.28	.8997	1.63	.9485	1.98	.9762
1.29	.9015	1.64	.9495	1.99	.9767
1.30	.9032	1.65	.9505	2.00	.9773
1.31	.9049	1.66	.9515	2.01	.9778
1.32	.9066	1.67	.9525	2.02	.9783
1.33	.9082	1.68	.9535	2.03	.9788
1.34	.9099	1.69	.9545	2.04	.9793

(Continued)

Table II. (Continued) Table of the Normal Distribution

$\frac{\mu}{\sigma}$	Reliability	$\frac{\mu}{\sigma}$	Reliability	$\frac{\mu}{\sigma}$	Reliability
2.05	.9798	2.40	.9918	2.75	.9970
2.06	.9803	2.41	.9920	2.76	.9971
2.07	.9808	2.42	.9922	2.77	.9972
2.08	.9812	2.43	.9925	2.78	.9973
2.09	.9817	2.44	.9927	2.79	.9974
2.10	.9821	2.45	.9929	2.80	.9974
2.11	.9826	2.46	.9931	2.81	.9975
2.12	.9830	2.47	.9932	2.82	.9976
2.13	.9834	2.48	.9934	2.83	.9977
2.14	.9838	2.49	.9936	2.84	.9977
2.15	.9842	2.50	.9938	2.85	.9978
2.16	.9846	2.51	.9940	2.86	.9979
2.17	.9850	2.52	.9941	2.87	.9980
2.18	.9854	2.53	.9943	2.88	.9980
2.19	.9857	2.54	.9945	2.89	.9981
2.20	.9861	2.55	.9946	2.90	.9981
2.21	.9865	2.56	.9948	2.91	.9982
2.22	.9868	2.57	.9949	2.92	.9983
2.23	.9871	2.58	.9951	2.93	.9983
2.24	.9875	2.59	.9952	2.94	.9984
2.25	.9878	2.60	.9953	2.95	.9984
2.26	.9881	2.61	.9955	2.96	.9985
2.27	.9884	2.62	.9956	2.97	.9985
2.28	.9887	2.63	.9957	2.98	.9986
2.29	.9890	2.64	.9959	2.99	.9986
2.30	.9893	2.65	.9960	3.00	.9987
2.31	.9896	2.66	.9961	3.01	.9987
2.32	.9898	2.67	.9962	3.02	.9987
2.33	.9901	2.68	.9963	3.03	.9988
2.34	.9904	2.69	.9964	3.04	.9988
2.35	.9906	2.70	.9965	3.05	.9989
2.36	.9909	2.71	.9966	3.06	.9989
2.37	.9911	2.72	.9967	3.07	.9989
2.38	.9913	2.73	.9968	3.08	.9990
2.39	.9916	2.74	.9969	3.09	.9990

(Continued)

Table II. (Continued) Table of the Normal Distribution

$\frac{\mu}{\sigma}$	Reliability	$\frac{\mu}{\sigma}$	Reliability	$\frac{\mu}{\sigma}$	Reliability
3.10	.9990	3.45	.9997	3.80	.9999
3.11	.9991	3.46	.9997	3.81	.9999
3.12	.9991	3.47	.9997	3.82	.9999
3.13	.9991	3.48	.9998	3.83	.9999
3.14	.9992	3.49	.9998	3.84	.9999
3.15	.9992	3.50	.9998	3.85	.9999
3.16	.9992	3.51	.9998	3.86	.9999
3.17	.9992	3.52	.9998	3.87 and up	.9999+
3.18	.9993	3.53	.9998		
3.19	.9993	3.54	.9998		
3.20	.9993	3.55	.9998		
3.21	.9993	3.56	.9998		
3.22	.9994	3.57	.9998		
3.23	.9994	3.58	.9998		
3.24	.9994	3.59	.9998		
3.25	.9994	3.60	.9998		
3.26	.9994	3.61	.9999		
3.27	.9995	3.62	.9999		
3.28	.9995	3.63	.9999		
3.29	.9995	3.64	.9999		
3.30	.9995	3.65	.9999		
3.31	.9995	3.66	.9999		
3.32	.9996	3.67	.9999		
3.33	.9996	3.68	.9999		
3.34	.9996	3.69	.9999		
3.35	.9996	3.70	.9999		
3.36	.9996	3.71	.9999		
3.37	.9996	3.72	.9999		
3.38	.9996	3.73	.9999		
3.39	.9997	3.74	.9999		
3.40	.9997	3.75	.9999		
3.41	.9997	3.76	.9999		
3.42	.9997	3.77	.9999		
3.43	.9997	3.78	.9999		
3.44	.9997	3.79	.9999		

Of course, the use of this procedure assumes that the load is equally divided among all the suspension lines. No data are available at the present time with which to verify this assumption. However, a test program to determine the distribution of loads in the suspension lines of a heavy duty parachute canopy is now being planned by WADD (May 1960). Should the results of this study indicate a significantly uneven load distribution, the maximum expected load per line should be used here.

The rationale upon which the above technique is based can be explained as follows. Let the break strength x be a random, normally distributed variable with mean, μ_x, and variance σ_x^2. Similarly, let the load y have mean, μ_y, and variance σ_y^2. Define the variable x - y as the difference between the mean break strength and the mean load. In terms of x - y, success or failure of a particular mission is expressed by $x - y \geq 0$, or $x - y < 0$. Hence the probability of success is simply the probability that $x - y \geq 0$.

The values of x - y are normally distributed with mean $\mu = \mu_x - \mu_y$ and variance $\sigma^2 = \sigma_x^2 + \sigma_y^2$. Hence the estimator $\bar{x} - \bar{y}$ may be used for μ and $s_x^2 + s_y^2$ for σ^2. Using the Non-Central t-Distribution, the probability that $x - y \geq 0$ can be estimated. Appendix C presents further details on the mathematical basis for the method.

As an example of the application of this method, the calculation of the reliability of the suspension lines of a parachute canopy from the g-load data at opening shock and the break strength of the suspension line webbing may be considered.

It will be assumed that the accelerometer readings from a series of ten instrumented drop tests of a canopy with 72 suspension lines are available. For each drop the vector sum of the 3-axis accelerometer readings corresponding to the load at opening shock is computed (the square root of the sum of the squares of the three readings). The mean (average) acceleration is assumed to be 3.14 g's, with a standard deviation of 1.30 g's and a load weight of 6730 pounds.

The mean load per line is 294 pounds, with a standard deviation of 122 pounds per line. The webbing in the suspension lines, as a result of 30 break tests, is found to have a sample mean strength of 1110 pounds (after deducting 20% for sewing -- see below), with a sample standard deviation of 218 pounds. Thus, the data needed to compute X and f for use in the Non-Central t-Distribution Graphs (Figure 4, 5, and 6) are:

Material		Load per Line	
$\bar{x}$	= 1110	$\bar{y}$	= 294
s_x^2	= 47,524	s_y^2	= 14,884
N_x	= 30	N_y	= 10
f_x	= 29	f_y	= 9

Since the variance of the load data is the smaller, the load data are used as s_1, N_1, and f_1. From equations 8, 9, 10, 11, 12 and 13 it is found that $\bar{N} = 20.3$, $f = 38.0$, and $X = 2.39$. Using these values in Figure 6, K is found to be 1.66 for a 99% confidence coefficient. From equation 14, $\mu/\sigma = 2.30$; using this value in Table II, the reliability is found to be .9893, with a confidence of 99%.

4.3.1.1 Effect of Seams on Materials Strength. At this point an additional factor affecting materials strength must also be considered; parachutes are constructed by sewing, and the seams have a definite effect upon the strength of fabric materials. Studies of the effects of seams on the strength of parachute webbing, cords and cloths (nylon) indicate that the seam weakens the sewed member by about 20% overall.[12,13] Thus, the mean material strength should be reduced by 20% before the computation of failure probability is made.

4.3.1.2 Effect of Multiple Layers of Materials. Many parachute components are constructed of more than one layer of a single webbing or tape sewed together to increase strength. The mean strength of such a member may be taken as the sum of the strength of the individual portions, reduced by 20% to allow for the fact that they are sewed together.

The standard deviation of this combined strength will be greater than the standard deviation of the single member. In general, since the multiple layers of fabric in the component were probably taken from the same batch of material during parachute manufacture, it must be assumed that the standard deviations of the combined layers will tend to be equivalent to the standard deviation of the individual layers multiplied by the square root of the number of layers. In such a case, it can be shown that for N layers, each of standard deviation s and mean strength $\bar{x}$, the combined strength will be approximately $0.8 \cdot N \cdot \bar{x}$ (allowing for seams), with a standard deviation of $s\sqrt{N}$.

4.3.1.3 Other Factors Affecting Materials Strength. The strength of nylon fabrics is affected, to some extent, by environmental exposure to extremes of temperature and humidity, to sunlight, to fume-laden atmospheres, and by previous history of extreme stresses which cause fatigue.14/

In general, protracted and extreme environmental exposures are required to cause significant changes in the strength of the nylon webbings, tapes, cords, etc. used in heavy-duty parachute construction. The handling and packing requirements for such parachutes and the fact that the packed parachutes are sealed in containers, generally with dessicants, are all pointed toward avoiding such exposure. Thus, it is not believed necessary to make any allowance for fabric strength reduction due to exposure to environments, natural or induced, in computing component reliability.

Weakening of load-bearing fabric members by fatigue may be a problem with parachute systems which are used more than once. The heavy-duty parachute systems under consideration herein, however, are single use systems. The only possible mechanism by which fatigue of load-bearing fabric members may occur is the exposure to successive shocks during the deployment and filling process. Discussions with parachute and fabric engineers of the magnitude of such shocks in relation to the type of materials used and the canopy design factors indicate that the reduction of component strength by fatigue during these processes is not a factor which need be considered in the reliability study.*/

4.3.2 Use of Engineering Computations of Loads

If instrumented drop tests from which measured loads on the components under study can be determined are not available, engineering estimates of the expected load on the component under design deployment conditions can be used in the above methodology. In this case the estimated load is used as $\overline{y}$, the standard deviation of the materials sample (s_x) is used as s, the number of materials samples tested (N_x) is used as N, and N_x-1 as f to calculate X from equations 6 and 13. K is found as above, and the f and N values used to find μ/σ from equation 14.

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*/ Personal Communication, WADD Parachute Branch and Materials Laboratory.

It should be noted that the confidence coefficient associated with the reliability calculated in this manner refers to the results of the fabric sample and computation process only, and does not reflect the adequacy of the engineering estimate of the loads. The confidence which may be placed in the accuracy of the load estimate can best be determined by the engineering group making the estimate. It is not possible to reflect this degree of confidence in the statistical confidence coefficient. The latter can only operate on the assumption that the engineering estimate is "good", and expresses the degree of confidence which may be placed in the reliability figure which results from the acceptance of the engineering calculation.

Methods for the engineering computations of expected loads are best chosen by the parachute design engineer; the application of engineering principles to parachute design analysis is beyond the scope of this study. Several books on the subject are available.[14,15]

4.4 Computation of Overall Reliability

The overall reliability, R, is the product of all the component reliabilities and the operational reliability. However, to arrive at an overall reliability value with an associated confidence coefficient, it is necessary to consider the confidence coefficients of the component and operational reliabilities as well.

The exact computation of the overall confidence coefficient is a rather complex matter, and one which, in general, is almost impossible without major computational effort. However, the overall confidence coefficient may be approximated by the product of the confidence coefficients of the individual reliability terms with sufficient accuracy for the purposes of parachute reliability evaluation.

To demonstrate the degree of approximation involved in this process, a system with only two components, C_1 and C_2, in series may be considered. Let the respective reliability estimates be R_{p_1,g_1} and R_{p_2,g_2}. This means that

$$\text{prob (true } R_1 \geqslant R_{p_1,g_1}) \geqslant g_1, \text{ and prob (true } R_2 \geqslant R_{p_2,g_2}) \geqslant g_2. \qquad (15)$$

Now the true overall reliability, R_g, is equal to true R_1 multiplied by true R_2. Since the event

$$E_3 \equiv (\text{true } R_g \geqslant R_{p_1,g_1} \cdot R_{p_2,g_2}) \qquad (16)$$

contains but is not necessarily contained in the product of the two events

$$E_1 \equiv \left(\text{true } R_1 \geqslant R_{p_1,g_1}\right) \quad \text{and } E_2 \equiv \left(\text{true } R_2 \geqslant R_{p_2,g_2}\right), \tag{17}$$

it follows that

$$p(E_3) \geqslant p(E_1)\, p(E_2) \geqslant g_1 g_2. \tag{18}$$

Since E_3 can occur when not both E_1 and E_2 occur, this means that $g_1 g_2$ is an underestimate of $p(E_3)$

An example may clarify this. Suppose

$$\begin{aligned} &\text{prob}\left(\text{true } R_1 \geqslant .93\right) \geqslant .95 \\ &\text{prob}\left(\text{true } R_2 \geqslant .97\right) \geqslant .95. \end{aligned} \tag{19}$$

Then prob $\left(\text{true } R_g \geqslant (.93)\,(.97)\right)$ is surely $\geqslant (.95)^2$. However, if $(.95)^2$ is used as the combined confidence coefficient, in effect many favorable possibilities are overlooked, such as:

$$\text{true } R_1 = .92, \text{ true } R_2 = .99.$$

However, to take all such possibilities into account would involve an enormous amount of calculation by any techniques now known.

In order to compute the overall reliability to a preselected confidence coefficient, it is necessary to compute each component reliability, and the operational reliability, to a confidence coefficient which will give the desired overall confidence coefficient when the product is taken.

As was shown in a previous section, a confidence coefficient of 90% (or .90) is probably the most efficient level for this type of computation, considering the amount and types of data likely to be available. It is not necessary that the confidence coefficients for the individual component reliability terms be equal, but only that their product be the desired coefficient for the final result. However, if equal confidence coefficients are used for all component terms, the calculations will be facilitated.

In general, if there are N terms in the model (one operational term and N-1 component terms), each of the individual reliability terms should be computed at a confidence coefficient approximating the Nth root of the desired final confidence

coefficient. The following table provides approximate values of the confidence coefficient required for the component terms to achieve a confidence coefficient of .90 in the final result:

No. of Component Terms	2	3	4	5,6	7,8,9	10,11,12
Confidence Coefficient	.95	.965	.975	.98	.987	.99

To avoid extreme complexity in the evaluation of the reliability model, it is recommended that the number of component reliability terms be held to the minimum which will give adequate results (probably not more than 12 terms, and generally less).

4.4.1 Parallel Components

In the previous discussion, all component reliabilities considered have been for components which are in series from a probability viewpoint. That is, the failure of any one component will result in the failure of the complete system. Parachute systems may also contain components in parallel. In this case more than one component with the same function is present, and the successful operation of any one component will result in successful operation of the entire system (provided, of course, no other failures occur).

An example of this is afforded by parachutes with two (or more) reefing line cutters, where operation of any one cutter is sufficient for successful disreefing. On the basis of empirical studies, suppose that the reliability of a single cutter is determined to be $R_{c,g}$ with confidence coefficient g. Then assuming independence of the cutters, the reliability of two cutters will be

$$1 - (1 - R_{c,g})^2 \qquad (20)$$

at a confidence coefficient of g. Thus, if $R_{c,.95} = .92$, the two cutters will have a reliability $1 - (1 - .92)^2 = 1 - .0064 = .9936$ with confidence coefficient .95.

In general, for any number, N, of parallel components, the reliability of the parallel system R_b will be

$$R_b = 1 - (1-R_{b_1 g_1})(1-R_{b_2 g_2})\ldots(1-R_{b_N g_N}) \qquad (21)$$

If $g_1 = g_2 = \ldots = g_N$, then the overall confidence coefficient will be g_1. If the confidence coefficients of the individual

reliability terms are not equal, the overall confidence coefficient will be that of the lowest term.

4.4.2 Canopy Clusters

Clusters of parachute canopies used to decelerate a single load are composed of components in parallel from a physical viewpoint. From a probabilistic viewpoint, however, their treatment depends on the design of the parachute system. If the system can operate successfully only with all canopies in the cluster successfully deployed, then each canopy is represented by a series term in the model, and the cluster is treated as a number of separate independent components. However, if the load can be decelerated successfully if one (or more) of the parachutes in the cluster fail, the situation is a series-parallel one from a probabilistic viewpoint. In general, the probability of failure of r identical canopies, p_r, out of a total of N in the cluster, when the probability of failure of a single canopy is p, is:

$$p_r = \frac{N!}{r!\ (N-r)!}\ p^r\ (1-p)^{N-r} \tag{22}$$

If m is chosen as the maximum number of canopies which can fail without affecting the success of the mission, then the probability of failure of the entire cluster, p_d, will be

$$p_d = \sum_{r=m+1}^{N} p_r \tag{23}$$

For the usual case of three identical heavy-duty parachute canopies in a cluster, if the cluster can operate successfully with only two of the canopies deployed, then the probability desired is p_d with m=1, which reduces to

$$p_d = 3\ p^2\ (1-p) + p^3 \tag{24}$$

The above discussions are based on the assumption that there is no interaction between the individual canopies in the cluster, that is, that the probability of failure of any one canopy in the cluster is no different from the probability of failure of the same canopy when used singly under the same deployment conditions. This assumption is difficult to prove.

However, analyses of the records of 799 standard cargo parachute drops using clusters and 599 single drops of the same types of parachutes reveals no significant difference in the canopy failure rates in the two cases, when failures due to rigging errors are eliminated. Since rigging errors are included in the packing error term in the reliability analysis of the heavy-duty parachute system, it is believed that the assumption of no interaction between cluster canopies for this type of parachute system is valid.

4.5 Application of Calculated Reliability Values

Obviously, the end result of the computation of the reliability of a parachute system by the methods described in this report can be used to evaluate the long-run performance which can be expected of large numbers of such systems. However, the process of reliability evaluation presented herein has broader, and possibly more valuable, applications.

In the process of evaluation of the reliability model, the components of the parachute system most likely to fail are evaluated individually, as is the effect of the expected human error rate in packing. These sub-results are really the key to the study of potential causes of failure in a parachute system, as well as a guide to the efficient expenditure of effort in the improvement of parachute system reliability.

To produce the most efficient parachute system for a given over-all cost, effort should be concentrated on achieving approximately the same degree of reliability for all components and for the packing process. Effort expended in this manner has the greatest payoff in increasing overall system reliability.

As was shown previously, the level of system reliability is influenced primarily by those components with the highest expected failure rates. Since the process of reliability analysis detailed herein detects these components explicitly, it can be of major value in locating those portions of a parachute system upon which the expenditure of further development effort will have the greatest effect on system reliability.

SECTION 5

EXAMPLE OF APPLICATION OF RELIABILITY ANALYSIS

To demonstrate the application of the methodology described in the previous section to the evaluation of parachute system reliability, a reliability analysis of a hypothetical heavy-duty parachute system is presented in detail below. While the actual parachute specifications used are not those of any specific system, they are representative of fairly complex operational heavy-duty parachute systems actually in use.

5.1 Parachute System Description

The hypothetical parachute system consists of two canopies, and operates as a two-stage retardation system with an explosive disconnect mechanism to separate the first canopy from the load after it has performed its function, allowing the second canopy to deploy. The system is primarily intended for relatively low-altitude, moderate-speed deployment -- about 500 knots below 10,000 feet. The initial weight of the drop unit, including the major canopies, pilot parachutes, deployment bags, load, etc. is 7000 pounds. The deployment of the system from a compartment in the end of the load is initiated by a static line, which removes the compartment cover to which the pilot chute deployment bag is attached.

5.1.1 First Stage Canopy

The first stage consists of a ribbon canopy with nominal diameter of approximately 20 feet. It is packed in a deployment bag with a small pilot chute attached to remove it from the compartment in the load and allow lines-first deployment. Weight of the canopy, deployment bag, pilot chute, compartment cover and static line is 140 pounds. The canopy is reefed by a skirt reefing line six feet in diameter; disreefing is effected by two M2A1 Reefing Line cutters with four second delays.

The canopy has 24 suspension lines made of nylon webbing, Type XVIII, MIL-W-4088C. There is a sewn loop at the lower end of each line which is attached to a corresponding lug on the periphery of the parachute compartment end of the load.

The mechanism which disconnects the canopy from the load consists of a length of primacord threaded through each of the suspension line lower loops in a channel in the circle of lugs. The primacord also passes through two 10 second delay train initiating devices consisting essentially of M2A1 reefing line cutters with their firing wires reversed and the cutter knives replaced with booster explosive charges. The delay trains are initiated at line stretch by lanyards attached to the canopy suspension lines. When the delay train ignites the booster charge, firing the primacord and separating the first canopy from the load, loop attachments on the suspension lines withdraw the deployment bag containing the second stage canopy and initiate its deployment.

5.1.2 Second Stage Canopy

The second stage of the system consists of a solid flat circular canopy 70 feet in diameter. It is packed in a deployment bag with a small quick-opening stabilization chute attached to its apex. It is deployed lines-first, followed by the canopy, and then by the stabilization chute. The latter keeps the canopy strung out in a straight line during filling. The canopy is reefed by a 10 foot diameter skirt reefing line; disreefing is effected by two M2A1 reefing line cutters with two second delays. Weight of the canopy, deployment bag, and stabilization chute is 130 pounds.

The 72 suspension lines of the canopy are made of 1/2" tubular nylon webbing, MIL-W-005625C. These terminate in sewn loops at their lower ends, and are attached to connector links in groups of six. Each of the connector links is attached to the upper end of a riser (twelve in all) made of Type XVIII nylon webbing, MIL-W-4088C. The lower end of the risers are attached to a second set of attachment lugs on the periphery of the end of a load, well separated from the lugs provided for attachment of the first canopy to avoid damage when the disconnect primacord is fired.

5.2 Preliminary System Analysis

It is assumed that the above described parachute system has been designed, several models have been built and drop tested, that design deficiencies have been rectified, and that at least one more series of development drop test have been run to prove the design. Final pre-production models of the system have been produced, and eleven instrumented drop tests conducted.

Study of these test results indicates that the critical components of the system from a reliability viewpoint (i.e., those which are most likely to fail) are the suspension lines, reefing line cutters, and delay device in the disconnect mechanism of the first stage canopy, and the suspension lines, reefing line cutter, and risers of the second stage canopy. Accordingly, these are the components to be subjected to reliability analysis.

5.3 Drop Test Data

The final series of eleven drop tests are assumed to be instrumented with 3-axis accelerometers, located on the load, which record the g-loads developed by the snatch force at line stretch, the opening shock to reefed condition, and the opening shock at disreefing for each of the two canopies. Table III presents the hypothetical g-loads computed from the resolution of the three instrument readings for each shock recorded (these are vector sums -- the square root of the sum of the squares of the longitudinal, lateral, and vertical accelerometer readings).

Examination of the distribution of the g-values indicates that they are approximately normal. The release speeds and altitudes (450-525 knots, 6000-10,000 feet) are close to the parachute system design limits of 500 kts and approximately 10,000 feet.

5.4 Suspension Line and Riser Reliability Analysis

To determine the loads to which each suspension line and each riser will be subjected, it is first necessary to calculate the mean load, and its standard deviation, for each of the three shocks on each canopy. Since all the drop units have been assumed to have the same weight, the computations are simplified if the means and standard deviations of the loads are first calculated in terms of g's and then converted to pounds by multiplying by the drop unit weight at the time of the shock. (If drop unit weights were not equal it would be necessary to convert to pounds by multiplying weight by acceleration prior to computation.)

The mean is the average -- the sum of the forces divided by the number of observations. The standard deviation is most readily computed from a formula developed from equation (5) of Section 4:

$$s = \sqrt{\frac{N \Sigma x^2 - (\Sigma x)^2}{N(N-1)}} \tag{25}$$

Table III. Hypothetical Drop Test Data

Drop No.	Canopy # 1 Snatch Force, g	Canopy # 1 Opening Shock, g Reefed	Canopy # 1 Opening Shock, g Disreefed	Canopy # 2 Snatch Force, g	Canopy # 2 Opening Shock, g Reefed	Canopy # 2 Opening Shock, g Disreefed
1	4.1	7.3	7.6	1.9	2.4	5.9
2	6.8	5.6	7.2	4.1	4.3	1.7
3	-	9.9	7.9	4.1	-	3.3
4	3.9	4.2	6.8	5.2	5.5	2.8
5	4.7	7.1	4.6	2.6	3.7	2.0
6	5.3	7.2	7.1	2.9	1.3	3.5
7	7.8	8.7	6.9	3.2	3.3	2.1
8	7.3	9.4	8.1	3.4	4.1	4.6
9	5.7	6.7	5.8	1.7	1.7	1.6
10	4.9	6.2	-	2.8	3.0	3.1
11	2.9	3.3	9.2	3.7	2.1	2.7

All Data: 450-525 kts release speed; 6000-10,000 ft. release altitude

The drop unit weight at release was given as 7000 pounds. The first canopy, with its deployment bag, cover, etc. weighs 140 pounds; thus the load weight during first canopy deployment is 6860 pounds. The second canopy with its bag, etc. weighs 130 pounds, reducing the load weight to 6730 pounds at its deployment.

To compute the load per line and the standard deviation of the load per line for each canopy, the appropriate total mean and standard deviation are divided by the number of lines, 24 for the first canopy, and 72 for the second. The load and its standard deviation per riser for the second canopy are found similarly, by dividing the total values by the number of risers (12). The results of these calculations are summarized in Table IV.

Materials test data are required on two types of webbings: Type XVIII of MIL-W-4088C (canopy #1 suspension lines and canopy #2 risers) and 1/2" tubular webbing of MIL-W-005625C (canopy #2 suspension lines). Table IX of Appendix B indicates that the mean tensile strength of the Type XVIII webbing is 7486 pounds, and the strength of the 1/2" tubular webbing is 1387 pounds. Since both of these materials are sewed in the process of manufacturing the canopies and risers, the tensile strength values are reduced by 20% to allow for loss of strength in seams (see paragraph 4.3.1.1 in Section 4). Thus, the materials strength data required for the computations are:

	Type XVIII	1/2" Tubular
Mean ($\overline{x}$)	5989 lbs.	1110 lbs.
Standard Deviation (s_x)	272 lbs.	218 lbs.
Number of Tests (N_x)	61	30
Degrees of Freedom (f_x)	60	29

The first step in the reliability analysis is the selection of the set of data to be used as s_1^2, N_1 and f_1, on the basis of the smaller variance (standard deviation squared) for each case to be considered. In Table V the data necessary for the evaluation of equations 6 through 11 of paragraph 4.3.1 in the computation of N, f, $\overline{x} - \overline{y}$, and s, are listed for each of the nine cases, along with the results of each computation.

Table IV. Computations of Mean and Standard Deviation, Loads per Line and per Riser

	Canopy # 1			Canopy # 2		
	Snatch	Opening Shock		Snatch	Opening Shock	
Total Loads	Force	Reefed	Disreefed	Force	Reefed	Disreefed
Mean (g's)	5.34	6.87	7.12	3.24	3.14	3.03
Standard Deviation (g's)	1.42	2.03	1.26	1.02	1.30	1.30
Mean (lbs)	36632	47128	48843	21805	21132	20392
Standard Deviation (lbs)	9741	13925	8643	6865	8749	8749
Load/Line						
Mean (lbs)($\bar{y}$)	1526	1964	2035	303	294	283
Standard Deviation (lbs) (s_y)	406	580	360	95.3	122	121.5
Load/Riser						
Mean (lbs) ($\bar{y}$)	-	-	-	1817	1761	1699
Standard Deviation (lbs) (s_y)	-	-	-	572	729	729
Number of Tests (N_y)	10	11	10	11	10	11
Degrees of Freedom (f_y)	9	10	9	10	9	10

Table V. Data Required, and Calculations of N, f, s^2, and $\bar{x}-\bar{y}$

Canopy #1 Suspension Lines, Snatch Force

$\bar{x} = 5989$	$\bar{y} = 1526$	$\bar{x}-\bar{y} = 4463$
$s_1^2 = 73984$	$s_2^2 = 164863$	$s = 488.7$
$N_1 = 61$	$N_2 = 10$	$N = 13.50$
$f_1 = 60$	$f_2 = 9$	$f = 18.34$

Canopy #1 Suspension Lines, Opening Shock, Reefed

$\bar{x} = 5989$	$\bar{y} = 1964$	$\bar{x}-\bar{y} = 4025$
$s_1^2 = 73984$	$s_2^2 = 336400$	$s = 640.61$
$N_1 = 61$	$N_2 = 11$	$N = 12.91$
$f_1 = 60$	$f_2 = 10$	$f = 14.76$

Canopy #1 Suspension Lines, Opening Shock, Disreefed

$\bar{x} = 5989$	$\bar{y} = 2035$	$\bar{x}-\bar{y} = 3954$
$s_1^2 = 73984$	$s_2^2 = 129600$	$s = 451.20$
$N_1 = 61$	$N_2 = 10$	$N = 14.36$
$f_1 = 60$	$f_2 = 9$	$f = 21.17$

Canopy #2 Suspension Lines, Snatch Force

$\bar{y} = 303$	$\bar{x} = 1110$	$\bar{x}-\bar{y} = 807$
$s_1^2 = 9082$	$s_2^2 = 47524$	$s = 237.92$
$N_1 = 11$	$N_2 = 30$	$N = 23.49$
$f_1 = 10$	$f_2 = 29$	$f = 37.20$

(Continued)

Table V. (Continued) Data Required, and Calculation of N, f, s^2 and $\overline{x}$

Canopy #2 Suspension Lines, Opening Shock, Reefed

$\overline{y} = 294$	$\overline{x} = 1110$	$\overline{x}-\overline{y} = 816$
$s_1^2 = 14884$	$s_2^2 = 47524$	$s = 249.81$
$N_1 = 10$	$N_2 = 30$	$N = 20.31$
$f_1 = 9$	$f_2 = 29$	$f = 38.00$

Canopy #2 Suspension Lines, Opening Shock, Disreefed

$\overline{y} = 283$	$\overline{x} = 1110$	$\overline{x}-\overline{y} = 827$
$s_1^2 = 14762$	$s_2^2 = 47524$	$s = 249.57$
$N_1 = 11$	$N_2 = 30$	$N = 21.28$
$f_1 = 10$	$f_2 = 29$	$f = 38.92$

Canopy #2 Risers, Snatch Force

$\overline{x} = 5989$	$\overline{y} = 1817$	$\overline{x}-\overline{y} = 4172$
$s_1^2 = 73984$	$s_2^2 = 327184$	$s = 633.37$
$N_1 = 61$	$N_2 = 11$	$N = 12.96$
$f_1 = 60$	$f_2 = 10$	$f = 14.91$

Canopy #2 Risers, Opening Shock, Reefed

$\overline{x} = 5989$	$\overline{y} = 1761$	$\overline{x}-\overline{y} = 4228$
$s_1^2 = 73984$	$s_2^2 = 531441$	$s = 778.09$
$N_1 = 61$	$N_2 = 10$	$N = 11.14$
$f_1 = 60$	$f_2 = 9$	$f = 11.65$

(Continued)

Table V. (Continued) Data Required, and Calculation of N, f, s^2 and $\bar{x}$

Canopy #2 Risers, Opening Shock, Disreefed

$\bar{x} = 5989$	$\bar{y} = 1699$	$\bar{x}-\bar{y} = 4320$
$s_1^2 = 73984$	$s_2^2 = 531441$	$s = 778.09$
$N_1 = 61$	$N_2 = 11$	$N = 12.4993$
$f_1 = 60$	$f_2 = 10$	$f = 12.9362$

The next step in the computation is the evaluation of X for each case by use of equation (13), using the values for N, f, s and $\bar{x}-\bar{y}$ calculated above. Since there will be eight reliability terms in the final model, the desired confidence coefficient for each term is .987 (see below and paragraph 4.4). The Non-Central t-Distribution nomograms in Figures 4, 5, and 6 are available for .90, .95, and .99 confidence only. Thus it will be necessary to calculate the reliability for all three confidence levels and interpolate graphically to obtain the reliability at .987 confidence. (In those cases in which the reliability at either 99% or 95% confidence coefficient is found to be .9999+, it is not necessary to compute reliability at a lower confidence coefficient, since it will obviously also be .9999+.)

Entering each plot of the Non-Central t-Distribution with X and f, three values of K (one for each confidence coefficient) for use in equation (14) can be found. From K, N, and f, μ/σ is computed for each of the three confidence values. The table of the areas under the Normal Curve, Table II, gives the corresponding value of reliability (R_g) for each μ/σ value. Interpolation on the plot of R vs. g (figure 7) gives the reliability value to the desired confidence coefficient.

The results of the above computations for each of the nine canopy cases are summarized in Table VI. Examination of the last column in this table indicates that Canopy #1 suspension lines have their lowest reliability (.9998) during opening shock to the reefed condition. The lines of Canopy #2 have their lowest reliability (.9910) during both opening shocks, while the risers on this canopy are at their lowest reliability (.9986) during the opening shock to the reefed condition. The three reliability figures above, consequently, are used in the computation of the final reliability value in paragraph 5.8.

5.5 Reefing Line Cutter Reliability Analysis

Both canopies use the M2A1 reefing line cutter in pairs to effect disreefing after the first stage of filling. Since no data on the performance of this cutter under high acceleration, and no data on the acceleration experienced by the cutter in this application are available, the reliability figures (at 99% confidence) presented in Table II of Appendix B are used in the computations. The M2A1 cutter was found to have $R_{.987}$ = .985 for both two and four second delays in non-tropical environments. This value is substituted in equation (15) to compute the reliability of the pair of cutters; the result is $R_{.987}$ = .9998.

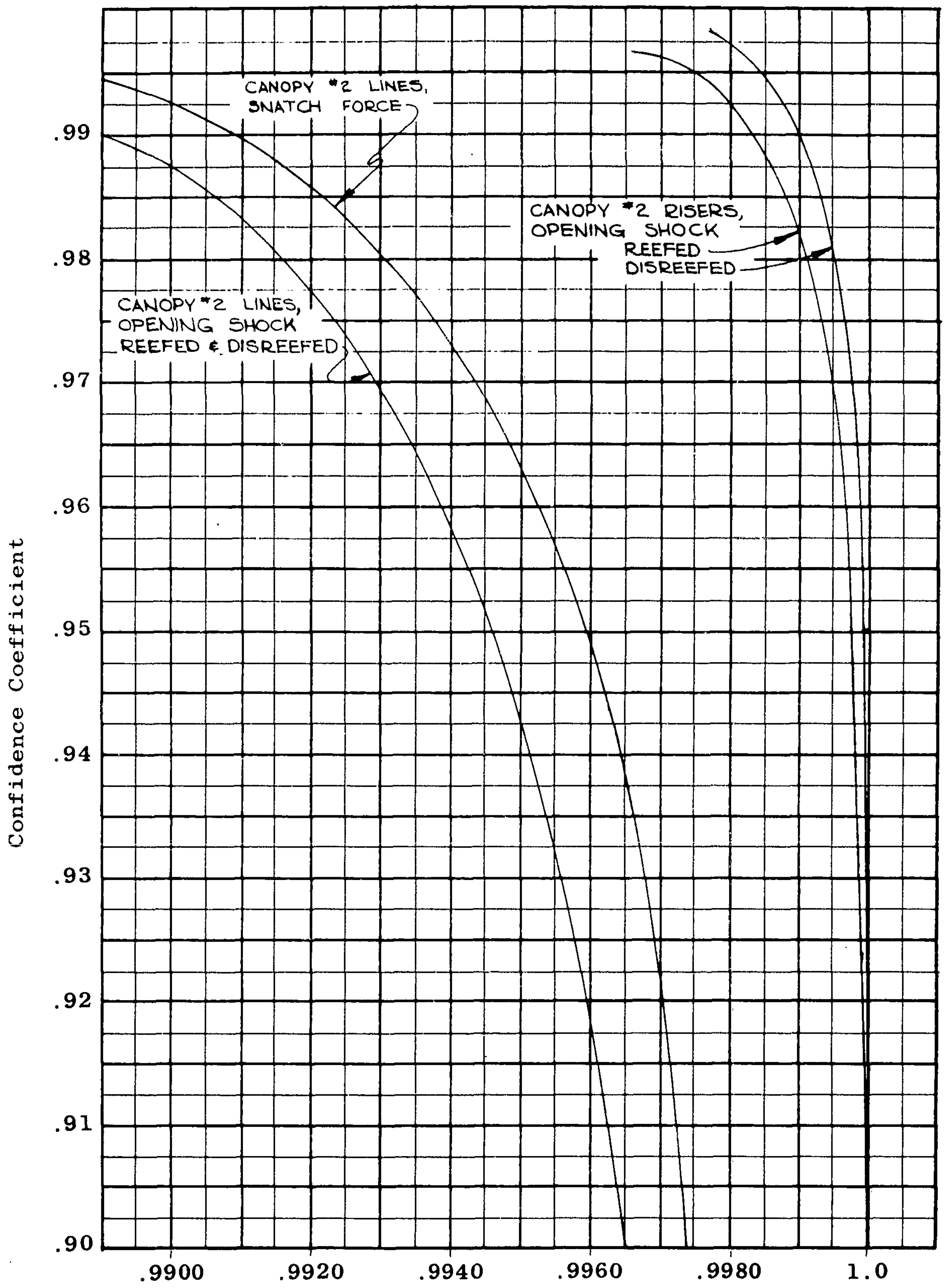

Figure 7. Interpolation for Reliability at Intermediate Confidence Coefficient

Table VI. Computation of Canopy Reliability

		Confidence = .90			Confidence = .95			Confidence = .99			
Canopy #1 Suspension Lines:	X	K	μ/σ	R	K	μ/σ	R	K	μ/σ	R	$R_{.987}$
Snatch Force	7.83	-	-	.9999+	-	-	.9999+	4.5+	> 5.39	.9999+	.9999+
Opening Shock Reefed	5.87	4.26	4.71	.9999+	3.87	4.28	.9999+	3.25	3.59	.9998	.9998
Opening Shock Disreefed	7.22	-	-	.9999+	-	-	.9999+	4.50	5.59	.9999+	.9999+
Canopy #2 Suspension Lines:											
Snatch Force	2.70	2.20	2.80	.9974	2.08	2.65	.9960	1.85	2.36	.9909	.9917
Opening Shock Reefed	2.39	1.95	2.70	.9965	1.84	2.55	.9946	1.65	2.29	.9890	.9910
Opening Shock Disreefed	2.42	1.98	2.71	.9966	1.86	2.55	.9943	1.67	2.29	.9890	.9910
Canopy #2 Risers:											
Snatch Force	6.14	-	-	.9999+	4.09	4.53	.9999+	3.44	3.81	.9999	.9999
Opening Shock Reefed	5.31	3.64	3.88	.9999+	3.27	3.48	.9998	2.74	2.92	.9983	.9986
Opening Shock Disreefed	5.46	3.86	4.08	.9999+	3.49	3.69	.9999	2.96	3.13	.9991	.9994

5.6 Disconnect Mechanism Reliability Analysis

The disconnect mechanism is essentially a pair of M2A1 reefing line cutters in parallel with the cutter knives removed (see paragraph 5.1.1). The reliability will, consequently, be the same as that of a pair of reefing line cutters in parallel $R_{.987} = .9998$.

5.7 Packing Reliability

Studies of cargo and extraction parachute performance in routine use (see Appendix A) indicate that an overall rate of about 2.7 failures per thousand canopies due to errors in packing have been experienced. Since the hypothetical parachute system considered herein has two canopies, there will be two packing reliability terms in the model. At a 98.7% confidence coefficient, the reliability value is .9953 for each canopy.

5.8 Overall Reliability

The reliability model for the hypothetical parachute system has eight terms, six of which are component terms: two canopy terms, one riser term, two reefing line cutter terms, and one disconnect mechanism term. The reason for the two operational terms is discussed above.

$$R = R_{p_1} \cdot R_{p_2} \cdot R_{c_1} \cdots R_{c_6} \tag{26}$$

Numerical values for each of these terms have been derived in the preceding paragraphs, and are summarized in Table VII, along with the computed component and operational reliability and the overall reliability.

5.9 Interpretation of Results

The interpretation of the overall reliability figure of .979 is evident: in the long run, about 21 failures per thousand uses can be expected with this hypothetical parachute system in at least 90% of all sets of trials.

However, the reliability values computed for the individual components are also of value in analyzing the system. For example, the suspension lines of Canopy #2 are obviously understrength for the loads to which they are subjected, despite the fact that the maximum of the three mean loads per line is only 303 pounds per line, while the material has a mean test tensile

Table VII. Component, Operational, and Overall Reliability

Component Reliability (R_c)

Canopy #1 Lines = .9998

Canopy #2 Lines = .9910

Canopy #2 Risers = .9986

Canopy #1 Reefing Line Cutters = .9998

Canopy #2 Reefing Line Cutters = .9998

Disconnect Mechanism = .9998

$$\text{Component Reliability} = \prod_{i=1}^{6} R_{c_i}$$

$$= .9998 \times .9910 \times .9986 \times .9998 \times .9998 \times .9998$$

Component Reliability = .9888

Operational Reliability (R_p)

Canopy #1 Packing = .9953

Canopy #2 Packing = .9953

$$\text{Operational Reliability} = R_{p_1} \cdot R_{p_2}$$

$$= .9953 \times .9953$$

Operational Reliability = .9906

Overall Reliability (90% confidence)

$$R = R_p \cdot R_c$$

$$= .9906 \times .9888$$

$$R = .979$$

of 1110 pounds after an allowance of 20% for seams. In this case the load is about 27% of the mean measured materials strength.

On the other hand, Canopy #1 appears to be highly reliable, with a maximum mean load per line of 2035 pounds and a materials mean strength (after sewing) of 5989 -- here the load is about 34% of the measured mean strength.

The explanation for this phenomenon lies in the relative magnitudes of the variances (standard deviation squared) of materials strength, tabled below for the two materials involved.

	Canopy #1 MIL-W-4088C, Type XVIII	Canopy #2 MIL-W-005625C,1/2"
Mean	5989	1110
Variance	73984	47524
$\frac{\text{Variance}}{\text{Mean}}$	12.4	42.8

As can be seen above, the ratio of variance to mean in the case of the suspension lines of Canopy #2 is several times that for Canopy #1, indicating a wider variability in the manufactured material strength. Study of the equations used in calculating the reliability of the canopy lines indicate that the final values are roughly proportional to variance (and hence strength variability). Thus, in the selection of materials for load-bearing members of parachutes, those materials with the lower variances (or standard deviations), will give better results, mean strengths being equivalent.

A second type of information may be derived from a comparison of the performance of the MIL-W-4088C Type XVIII webbing in the suspension lines of Canopy #1 and in the risers of Canopy #2. Data for the worst of the three shocks are tabulated below.

	Canopy #1 Lines	Canopy #2 Risers
Mean Load/Line or Riser	1 964	1 761
Variance/Line or Riser	336 400	531 441
$\frac{\text{Variance}}{\text{Mean}}$	171	302
Reliability	.9998	.9986

It can be seen that the webbing performs better in the suspension lines than in the risers, despite the fact that the load per member is higher in the case of the lines. Here, again, the variances are the key to the problem. The shocks on the risers are considerably more variable than the shocks on the lines, as indicated by the ratios of the variances to the means.

Since the parachute system analyzed herein is a hypothetical one, it is not really possible to find a physical cause for this variability. However, studies of drop test records of actual parachute systems similar to the hypothetical one indicate that the variability of the shocks is due primarily to instability of the load during the deployment and filling of the canopy. Thus, to increase the reliability of components affected by this instability it is necessary for the parachute designer to have some voice in the design of the load for better aerodynamic properties. Of course, some improvement may also be expected by increasing the stability of the canopy during the deployment and filling process.

The reliability analysis can also provide an indication of any required improvement in the reliability of auxiliary mechanical devices used in the parachute system. In the example, the reefing line cutter and the disconnect mechanism reliabilities are a case in point.

Taken singly, each of these devices has lower reliability (.985) than that calculated for the overall component reliability of the parachute system (.988). Since they are used in parallel pairs, however, the reliability of the two is high enough (.9998) so that their effect on overall system reliability is not serious. An increase in the reliability of each device from .985 to .993 would bring the reliability of the parallel pair to .9999+ (regarded in this computation as high enough to be omitted from the calculation), resulting in only a minor increase of overall system reliability (from .979 to .980). However, should the reliability of the auxiliary device be lower, an attempt at improvement of its performance is definitely indicated as a means of increasing overall system reliability without major redesign effort.

It must be realized that the parachute system analyzed in this section is a hypothetical one. No actual parachute system which has reached the point of final operational testing is likely to have reliability as low as that calculated for the example, either in its individual components or on an overall basis. The hypothetical system has had weak points deliberately designed into it to illustrate the computational methods and the types of conclusions which may be drawn from their results.

However, the causes of unreliability in the example are actually drawn from parallel cases in a number of operational parachute systems, and represent typical problems which may arise in heavy duty parachute reliability analysis.

APPENDIX A

PARACHUTE PACKING RELIABILITY DATA

Study of parachute drop records, and discussions with personnel of military units using large numbers of cargo parachutes, indicate that human error in the parachute packing process is a major source of parachute system failure. Thus, in developing a mathematical model and basic data for the evaluation of the reliability of heavy-duty parachute systems, detailed consideration of parachute failure rates due to packing errors was included.

The data available in sufficient quantity for a statistical approach to the study of parachute packing reliability result from drops of three parachute types: (1) Cargo parachute systems in routine field use, (2) Man-carrying parachutes used by airborne troops, and (3) Test data on experimental and development-phase parachute systems. Since the reliability model developed in this study is pointed toward the evaluation of the performance of heavy-duty parachute systems, the ideal data source would be packing error experience with parachutes of this type. Unfortunately, statistically adequate data for these operations were not available for this study, and it was necessary to use available information.

Of the three types of parachute use data available in quantity (see above), two, man-carrying and experimental-developmental uses, were rejected as being too unlike the data actually required for use in this study. Man-carrying parachutes are of almost completely different design and load capacity; experimental and development-phase parachute performance is so uncertain as to make the differentiation between packing failures, design failures, and other types of failures very uncertain. Thus, the choice falls upon cargo parachute systems as being the closest type to the heavy-duty systems available, as far as the packing problem is concerned.

Actually, information gathered during interviews with personnel working with heavy-duty parachutes and cargo parachutes indicates that the packing process for the former is probably done with somewhat greater care than the usual packing process for cargo parachutes, and that there is a possibility that the overall packing failure rates in the two cases may not be the same.

To base the study of packing failure rates on processes resembling heavy-duty parachute packing as closely as possible, the cargo parachutes studied were those used by test agencies developing and testing methods of use of standard cargo parachute systems, rather than testing the parachutes themselves or using them in tactical airborne operations. Data gathered in field interviews indicate that personnel packing the parachutes in these units would tend to have greater continuity of task assignment than their counterparts in tactical airborne units.

Furthermore, it is expected that these people would tend to exercise more care in the packing process than the field units, since the equipment to be test airdropped is generally available to them in very limited amount, making it essential that the chances of failure in the test from extraneous causes be held to a minimum. Since these test organizations use the standard cargo parachutes repeatedly as routine tools in developing airdrop methods, it can be expected that the level of proficiency in the packing process in these units will be as great as can be expected in any type of parachute use.

The actual data used in the analysis of packing failure rates were obtained chiefly from the project files of the Airborne Test Division of the U.S. Army Airborne and Electronics Board at Fort Bragg, North Carolina, and the Airborne Systems Test Division of the U.S. Army Quartermaster Field Evaluation Agency at the Yuma Test Station, Arizona. These data were supplemented by parallel types obtained in smaller amounts from the routine drop test activities of the 6511th Test Group (Parachute), NAAS, El Centro, California, and a few other agencies.

In all, records of 5563 uses of standard parachute systems were found. The failures recorded were studied carefully, generally after consultation with personnel of the organizations involved, to determine the number of failures which could be attributed to errors in the packing process (at the same time, data were collected on materials failures in the G-11A and G-12D Canopies for other analyses -- see below).

The data collected were analyzed for packing failure rates for two cargo parachute types, the G-11A and G-12D, and for all standard extraction parachutes as a group (combining 15, 22, 24 and 28 foot ring-slot and fist-ribbon types to obtain a sufficiently large sample). All other cargo parachute types have been grouped as "miscellaneous".

Packing failure causes included six cases in which reefing line cutters were not armed, one case in which the cut reefing line could not slip through the reefing rings to release the canopy skirt, one case in which the canopy was packed in such a manner that it emerged from the bag twisted, three cases in which improper cut knife installation prevented deployment, one case of the use of too strong a break cord, one case in which the canopy was not attached to the bridle, and two cases of static lines improperly attached to the pack. Thus, 14 of the 15 packing failures can be attributed to auxiliary devices (cutters, attachments, knives, etc.), with only one failure due to the canopy packing itself.

Parachute Type	No. of Uses	No. of Packing Failures	Observed Failure Rate
G-11A	1964	7	.00356
G-12D	1333	3	.00225
Extraction	1009	3	.00297
Misc.	1257	2	.00159
Combined	5563	15	.00270

Examination of the results of this study indicates that the observed failure rate is between approximately 1.6 and 3.6 per thousand parachute packings. While this rate is not based on use data for heavy-duty parachute systems, it appears to be the best estimate available until sufficient data on heavy-duty parachute uses are available for analysis.

Of course, before these data can be substituted in the reliability model, the confidence coefficient must be selected, and the expected failure rate computed from Equation 4 or from Table I. The combined values give reliabilities of .9962, .9958, and .9952 at confidence coefficients of 90, 95, and 99% respectively.

APPENDIX B

PARACHUTE COMPONENT RELIABILITY DATA

As can be seen from the discussion of the methods of evaluation of the reliability model presented in this report, a considerable body of data on the components of the parachute system are required to evaluate system reliability. Many of these data are unique to the specific system under evaluation, and must be obtained for the specific system. Some types of component reliability data, however, are applicable to a wide range of parachute systems. Where possible, these have been collected as part of this study. Such data on reefing line cutters, solid flat circular canopies, and some parachute materials are presented below in a form directly applicable to the evaluation of the reliability model.

B.1 Solid Circular Canopy Reliability

In the study of cargo parachute drop data used to evaluate parachute packing reliability (See Appendix A), data on material failures for the G-11A (100-foot) and G-12D (64-foot) cargo parachute canopies were assembled, inasmuch as these canopies are used as components of heavy-duty parachute systems. In the case of the G-11A canopy used under its design conditions, one failure attributable to materials failure (suspension lines broke) in 1964 uses were found. For the G-12D, no failures attributable to materials were found in the records of 1333 uses. Computed reliabilities for various confidence coefficients (from Table I, page 32) are presented below.

			Reliability				
	No. Uses	No. Failures	90%	95%	96.5%	98%	99%
G-11A	1964	1	.9980	.9976	.9974	.9969	.9966
G-12D	1333	0	.9983	.9977	.9975	.9969	.9966

These data are usable in their present form as component reliabilities in computations for systems in which the specific canopies are used under their design conditions. Computations based on larger samples, however, will provide better estimates of the reliabilities of these canopies. Thus, should more use data on these canopies become available, they should be combined with the above.

B.2 Reefing Line Cutter Reliability*/

Reefing line cutter field performance data based on actual parachute use are generally not available in a form from which reliability can readily be calculated. Since most parachute systems employing reefing lines have more than one cutter, and since the successful operation of any one cutter will disreef the canopy, disreefing failure data generated by field use are generally based on simultaneous failures of more than one cutter. Laboratory test data on single items are available in sufficient quantity, however, to allow computation of reliability estimates on some reefing line cutter models.

In the analysis of these test data to determine expected failure rates for reefing line cutters, the conditions of use of the cutter are very important factors in determining applicability of the reliability figures derived. The most commonly used reefing line cutter, the M2A1, was designed for use primarily in low-altitude, low-deployment-speed cargo parachute systems. The original specifications called for an operating altitude of 2000 to 5000 feet, (although the cutter was to be operable to 150,000 feet with broader tolerances) a temperature range of $70^{\circ} \pm 10^{\circ}F$, timing tolerance of $\pm$ 10% at $70^{\circ}F$ (later broadened somewhat), a pull angle of not more than $\pm 22.5^{\circ}$ off the longitudinal axis of the cutter, and a pull load to fire of 20 pounds (later changed to 60 inch pounds maximum to fire, not to fire at 10 pounds static pull). The cutter was not designed to be subject to high shock loadings during operation.*/

When used in a standard cargo parachute system (e.g. the G-11A) these conditions, with the possible exception of the timing tolerance due to varying operating temperature, are generally met. Temperature effects on timing are said to result in an additional 10% decrease in delay time at $+160^{\circ}F$ and an additional 20% increase at $-65^{\circ}F$.*/

Discussions with parachute using units and with Army Ordnance Corps personnel indicate that the performance of the M2A1 reefing line cutter in standard cargo parachutes (generally two are used to increase reliability) is satisfactory. Laboratory test results indicate comparatively low failure rates for this model:

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*/ Reefing line cutter test data presented below were obtained from the Industrial Engineering Division, Picatinny Arsenal.

Test Conditions	Total Tested	No. Good	No. Failed	Reliability (90% Confidence)
Laboratory Ambient (70°F)	1100	1100	0	.998
Low Temperature (-65°F)	690	688	2	.992
High Temperature (+165°F)	550	548	2	.990
High Temperature, 100% Relative Humidity	260	248	12	.931
Parachute Use Test (Cargo Parachute)	100	100	0	.977

With the exception of the combined high temperature-humidity test, the above results suggest that the standard M2A1 cutter, under static conditions, is a highly reliable device.

The high temperature-humidity environment does reduce this reliability to a considerable extent. This environment affects the reefing line cutter by causing distortion of the nylon ball which holds the firing pin in place until the firing wire is pulled. When the firing wire is removed from the cutter by the lanyard at the end of line-stretch during parachute deployment, the nylon ball is allowed to move back in a channel, releasing the firing pin so that the firing spring may drive it against the detonator. If this ball is distorted by high temperature and high humidity, it can not readily move back into the channel. The distortion does not always cause complete failure of the cutter, but often causes considerably lengthened delay times, since the ball is usually squeezed into the channel gradually by the force of the firing spring.*/

Since the high-temperature-humidity environment represents the effects of the probable extremes of a hot, moist tropical climate on the cutter[16/]; its effect need only be considered when extended tropic-based parachute missions are involved.

Since the reefing line cutter is installed on the skirt of the canopy, it will be subject to considerable acceleration forces during deployment and inflation. To represent the environmental conditions of reefing line cutter use more completely, it is necessary to superimpose the effects of these acceleration forces on cutter reliability on the above data. Extensive test data on the effects of acceleration forces on reefing line cutter performance are not available, although

*/ Personal Communication, Picatinny Arsenal.

an Air Force drop test program is now under way (May 1960) to develop information on the spectra of forces to be expected and the performance of standard reefing line cutters under high acceleration conditions.

In general, laboratory tests show that the acceleration forces which affect reefing line cutter performance are those which are applied during the firing pin operation and delay train burning. It appears that the explosive force of the charge which drives the knife through the reefing line during the actual cutting operation is sufficient to overcome any g-forces which may be encountered.

The direction of firing pin travel and of the movement of the flame front in the delay train during burning are the same. Laboratory testing at Picatinny Arsenal has indicated that acceleration forces applied in the direction of firing pin travel and flame front burning have virtually no effect on the reliability of the cutter; it works just as well under this type of acceleration as under static conditions. However, when the acceleration forces are applied in the opposite direction, the magnitude of the force seems to determine whether the cutter will work or not.

An acceleration force applied opposite to direction of firing pin travel tends to oppose the effect of the firing pin spring, often resulting in light strikes which do not fire the detonator. When the delay train is burning, the particular composition used in the M2A1 cutter burns with a flame front preceded by a very thin zone of liquified delay composition. An acceleration force in the direction opposite to the flame front travel tends to separate this liquified zone from the unmelted delay powder ahead of it, thus creating a gap in the delay train, and often preventing further combustion of the delay material. Of course, accelerations in the opposite direction, that is, in a direction assisting the firing pin spring, force the burning flame front into the unburned portions of the delay composition, and do not have these effects.

It should be noted that a centrifuge was used to apply the accelerations in these tests. Thus, the reefing line cutter was subjected to a steady-state type of acceleration, rather than the short-period, shock-type acceleration to be expected at the reefing line cutter pocket during parachute canopy deployment and filling. Thus, the results of the testing are indicative of qualitative effects, but cannot be used to predict quantitatively the effect of the shocks encountered in field use.

Figure 8 is a plot of the test data on the effects of counter-firing pin centrifuge acceleration forces on the M2A1 (and M2A1 Modified) reefing line cutter made available for this study by Picatinny Arsenal. Three curves and one point on this figure represent the performance of the M2A1 standard reefing line cutter under acceleration. Two difference delays are represented, and in the case of the two-second delay, two difference conditions of acceleration. In one case, the opposing acceleration forces were applied parallel to the direction of firing pin travel; in the other these opposing forces were applied at somewhat of an angle off the axis of the firing pin travel. As can be seen, the effects of opposing angled forces on reliability are somewhat greater than the effects of opposing parallel forces. This is to be expected, since the angled forces, in addition to affecting the force of the firing pin spring, add frictional forces by driving the firing pin against the sides of the cutter bore.

The four-second delay cutter is seen to be affected more by acceleration than the two-second cutter. This is attributed to the fact that the four-second delay line is composed of two two-second pellets, while the two-second cutter contains only one pellet. Thus, the acceleration forces on the four-second cutter may tend to pull the pellets apart slightly, creating a blank zone through which the flame front cannot travel.

In the operation of the reefing line cutter in the parachute system, it is generally not possible to determine whether the reefing line cutter firing pin travel and delay train burning are affected by acceleration forces parallel to or at an angle to the long axis of the cutter, although it would appear that the data on angled forces are probably more representative of the effects of acceleration on reefing line cutter reliability.

The magnitude of the forces on the reefing line cutter in any given parachute drop will depend on the specific parachute design, on the speed at which the parachute is deployed, and the load. Thus, to determine the reliability figure to be applied in a specific case, information on the g-forces expected and their orientation with respect to the reefing line cutter is required, either from instrumented field trials, or from engineering calculations. In addition, performance tests on the cutters under shock conditions, either laboratory or field, will also be required. The aforementioned Air Force test program should provide such data.

In field investigations of failures of the M2A1 standard reefing line cutter in high speed parachute applications, it was found that in many cases the acceleration forces applied to the cutter were occurring in the direction opposite to

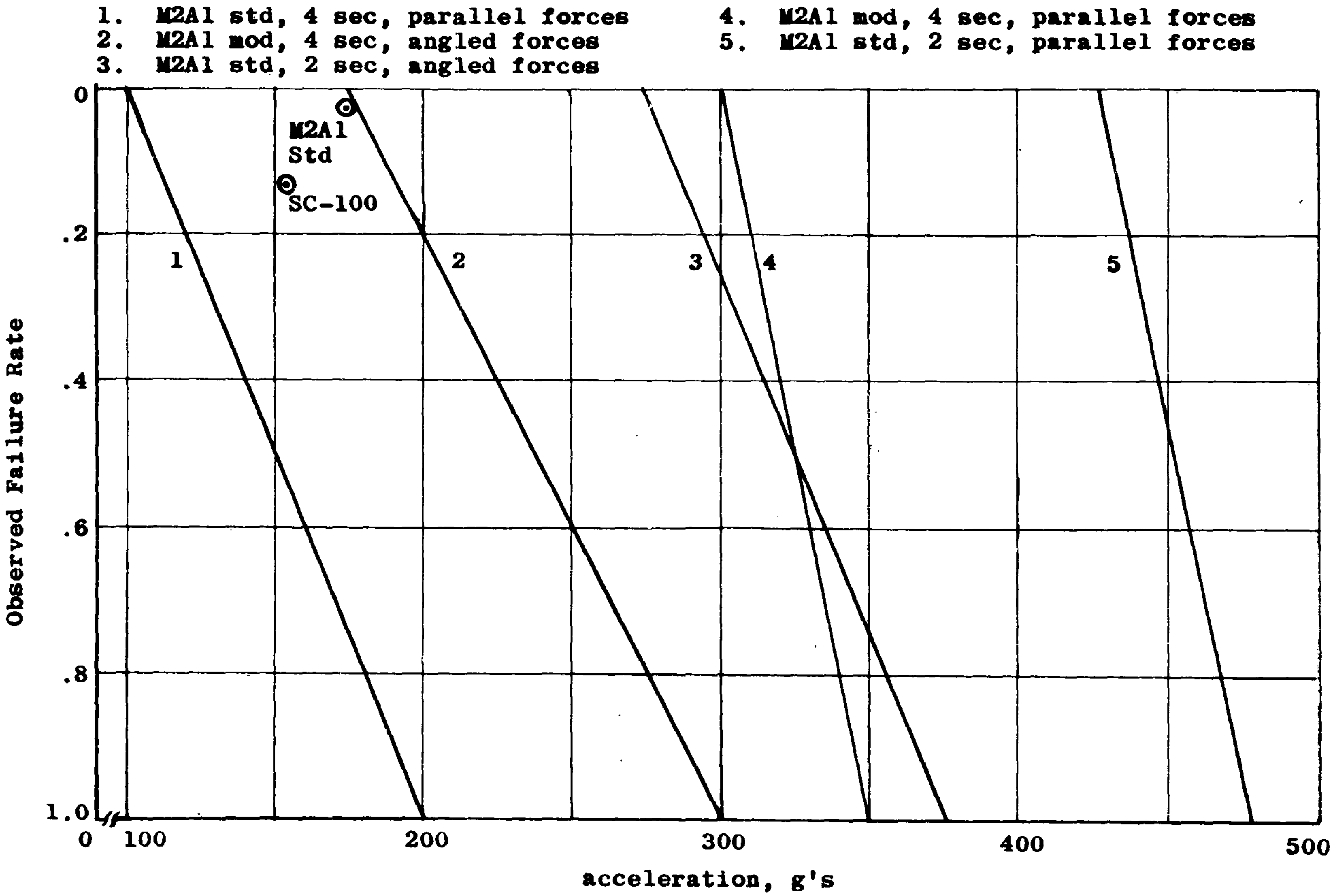

Figure 8. Effects of Reverse Acceleration on Reefing Line Cutters.

firing pin travel. In an attempt to eliminate the M2A1 reefing line cutter failure problem for this type of application, the modified M2A1 reefing line cutter was developed. This model is identical with the standard M2A1 except that it is installed in the opposite direction, and has a stronger firing pin spring. In order to effect installation in a reverse position, the direction of firing wire pull was reversed.

Figure 8 indicates that the increase in strength of the firing wire spring, in general, increases the upper limit of tolerable acceleration on the four-second cutter from the 100 g limit of the M2A1 standard to approximately 175 g's for the four-second M2A1 modified cutter with opposing accelerations applied at an angle. The M2A1 modified cutter may thus be assumed to have its static reliability to a somewhat higher degree of acceleration than the standard model.

Laboratory testing of the modified M2A1 cutter indicates that it is a relatively highly reliable device under static conditions:

Test Conditions	Total No. Tested	No. Good	No. Failed
Laboratory Ambient (70^{o}F)	400	400	0
Low Temperature (-65^{o}F)	80	80	0
Cycled High-Low Temperature, High Humidity	20	20	0
Total	500	500	0

Computed reliability, 90% confidence - 0.995

The M2 reefing line cutter was the original model from which the M2A1 was developed, and differs from it in minor details of construction. Field investigation indicates that a considerable number of M2 cutters are still in use, and that procurement of this model may be resumed.

Laboratory test data (Picatinny Arsenal) indicate relatively high reliability under static conditions:

Test Conditions	Total No. Tested	No. Good	No. Failed
Laboratory Ambient (70°F)	702	702	0
Low Temperature (-65 to -95°F)	260	260	0
High Temperature (+160°F)	30	30	0
High Temperature, High Humidity (+165°F, 100%)	120	120	0
Total	1112	1112	0

Computed Reliability 90% confidence - .998

Some laboratory test data are also available on two proprietary reefing line cutters, the SC-100, manufactured by Ordnance Specialties, Inc., El Monte, California, and the OA-A3, made by Ordnance Associates, South Pasadena, California:

SC-100

Test Conditions	Total No. Tested	No. Good	No. Failed
Laboratory Ambient (70°F)	500	500	0
Low Temperature (-65°F)	180	180	0
Cycled High-Low Temperature, High Humidity	20	20	0
Parachute Use Test	100	100	0
Total	800	800	0

Computed reliability, 90% confidence - 0.997.

In centrifuge acceleration tests at 154 g (with the forces opposing the direction of firing pin movement), 12 cutters failed of 100 tested.

OA-A3

Test Conditions	Total No. Tested	No. Good	No. Failed
Laboratory Ambient (70°F)	70	70	0
Low Temperature (-65°F)	40	40	0
Cycled High-Low Temperature, High Humidity	10	10	0
Total	120	120	0

Reliability, 90% confidence - 0.981

No data on acceleration effects on the OA-A3 were available for this study.

The laboratory test data, as interpreted in the light of user experience, indicate that the reliability values summarized in Table II, for the conditions noted, may be used for computational purposes in evaluating the reliability model. It should be noted that these are tentative values, and are subject to revision as more extensive test and use data are accumulated.

B.3 Strength of Parachute Materials

Data on the measured break strength of parachute materials from which the mean and standard deviation of materials strength can be calculated will be required to evaluate the reliability of many parachute components and subcomponents. Such data, in a form which is discussed below, are readily available in the files of parachute and parachute materials manufacturers, since webbings, tapes, cords, etc. are tested by both the textile manufacturer and the parachute fabricator to insure compliance with the strength requirements of the appropriate specifications.

The original break-test records of the following firms were made available for this study (all are materials manufacturers except for Pioneer Parachute Company):

Pioneer Parachute Company, Inc., Manchester, Conn.
Bally Ribbon Mills, Inc., Bally, Pa.
Phoenix Trimming Co., Chicago, Ill.
Narricot Corporation, Philadelphia, Pa.
Buser Silk Corporation, Paterson, N.J.
Essex Mills, Inc., Pawtucket, R.I.
Fox Specialty Co., Inc., Lowell, Mass.

Table VIII. Reefing Line Cutter Reliability

Type	Use Conditions	Trials	Failed	Reliability at Confidence Coefficient 90%	95%	98%	99%
M2A1 Std	Low g, Non-Tropical	550	2	.990	.989	.986	.985
M2A1 Std	Low g, Tropical	260	12	.931	.925	.916	.912
M2A1 Std	High g, with Firing Pin	550	2	.990	.989	.986	.985
M2	Low g, Non-Tropical	962	0	.998	.997	.996	.995
M2A1 Mod	Low g	500	0	.995	.994	.992	.991
M2A1 Mod	High g, with Firing Pin	500	0	.995	.994	.992	.991
SC-100	Low g	800	0	.997	.996	.995	.994
SC-100	High g, with Firing Pin	800	0	.997	.996	.995	.994
OA-A3	Low g	120	0	.981	.975	.966	.962

In addition, some records on tensile strength tests of high-strength materials were obtained from the Textile Branch of the WADD Materials Laboratory.

For most of the materials, test data were obtained on the products of more than one manufacturer; for those materials most frequently used in parachute construction data were available from three or four sources. All data on webbings and tapes represent production during 1959 and 1960, and thus reflect the use of the current type of nylon yarn. The results of four or five break tests on each batch of webbing and tape manufactured were available in practically every case. The mean of these was used as the break strength of the batch. In the cases of cords, one break per batch was obtained. Means and standard deviations were computed by usual statistical methods, and are presented in Table IX.

It is believed that the data collected on materials for which 30 or more batches were tested are adequate for the purposes of the reliability computations described in this report. If materials with fewer tests must be considered, it is recommended that further break test data be collected.

One additional comment on the above type of break strength data seems to be necessary. The break tests from which the data were derived were done on fabric tensile testing machines (Tinius Olsen, Scott Tensile Tester, and similar types) in which the rate of jaw separation is relatively low--- of the order of inches per minute. The loads to which these materials are subjected during the deployment of a parachute are applied at a much higher rate. Accelerometer traces indicate that the total elapsed time for both snatch force and opening shock in a typical parachute canopy is of the order of one second.[14]

Thus, in order for the break test data to be completely valid, it is necessary to establish a correspondence between tensile strength at low and high rates of load application. Very little data are available in this area. However, the results of a study performed by MIT for the Quartermaster Research and Development Center,[20] show that there is little difference in the break strength of nylon materials under loads applied at rates varying from two inches per minute to 48 feet per second. Discussions with the personnel of the Textile Branch of the WADD Materials Laboratory, and of the Textile, Clothing and Footwear Division of the Quartermaster Research and Engineering Command, indicate that other experience tends to confirm these results.

Thus, available evidence appears to warrant the use of laboratory tensile test results to represent the performance of the materials in the computation of parachute canopy reliability.

Table IX. Measured Mean and Standard Deviation of Break Strength, Nylon Parachute Webbings, Tapes and Cords*/

Specification	Type	Spec. Strength (lbs)	Mean Meas. Strength (lbs)	Standard Deviation (lbs)	No. of Tests
Webbings:					
MIL-W-4088C	I	500	576	27.8	118
"	II	600	796	97.9	76
"	III	800	976	84.4	46
"	IV	1800	2461	39.7	10
"	V	3000	3135	76.6	6
"	VI	2200	2868	89.9	62
"	VII	5000	6659	181	11
"	VIII	3600	4492	171	79
"	IX	8200	no data		
"	X	8700	9502	307	16
"	XI	2400	3168	132	23
"	XII	1200	1527	145	47
"	XIII	6000	7893	410	48
"	XIV	1200	1963	21.7	5
"	XV	1500	2861	94.2	6

*/ Data presented are for undyed (natural) and condition "U" materials.

Table IX. (Continued) Measured Mean and Standard Deviation of Break Strength, Nylon Parachute Webbings, Tapes and Cords

Specification	Type	Spec. Strength (lbs)	Mean Meas. Strength (lbs)	Standard Deviation (lbs)	No. of Tests
Webbings (cont):					
MIL-W-4088C	XVI	4500	5262	321	5
"	XVII	2400	3159	88.7	20
"	XVIII	6000	7486	272	61
"	XIX	10,000	11,699	1563	23
"	XX	9000	10,016	461	30
"	XXI	3600	4216	137	6
MIL-W-4088D	XXII	?	8556	36	6
MIL-W-005625C	1/2"	1000	1387	218	30
"	9/16"	1500	1855	119	30
"	5/8"	1850	2095	171	5
"	3/4"	2300	3019	98.7	19
"	1"	3500	4311	181	69
MIL-W-5666	1-5/16"	500	630	23.7	6
MIL-T-005038B	IV-1"	1000	1209	49.1	76
"	IV-1-1/2"	1500	1773	107	63

Table IX. (Continued) Measured Mean and Standard Deviation of Break Strength, Nylon Parachute Webbings, Tapes and Cords

Specification	Type	Spec. Strength (lbs)	Mean Meas. Strength (lbs)	Standard Deviation (lbs)	No. of Tests
Tapes:					
MIL-T-005038B	II-1"	900	1058	77.4	16
"	III-3/8"	200	226	5.6	8
"	III-1/2"	250	282	16.6	37
"	III-3/4"	400	466	15.9	73
"	III-1"	525	605	32.0	18
"	V-9/16"	500	550	19.0	27
"	VI-3/4"	425	478	24.5	5
MIL-T-5608E	clB-III	70	73.7	1.0	21
"	clC-III	90	96.9	2.9	70
"	clC-V	300	317	25.7	169
"	clD-II	460	477	18.2	23
"	clE-II	1000	1173	36.1	73
"	clE-III	1500	1727	43.1	7
"	clE-IV	2000	2314	109	7
"	clE-V	3000	3361	12	7

Table IX. (Continued) Measured Mean and Standard Deviation of Break Strength, Nylon Parachute Webbings, Tapes and Cords

Specification	Type	Spec. Strength (lbs)	Mean Meas. Strength (lbs)	Standard Deviation (lbs)	No. of Tests
Tapes (cont):					
MIL-T-6134A	1"	525	646	34.3	117
Cords:					
MIL-C-5040B	I	100	121	3.98	68
"	IA	100	140	7.53	128
"	II	375	440	19.6	125
"	III	550	603	11.2	192
"	IV	750	771	16.5	52

APPENDIX C

CONFIDENCE INTERVALS ON THE PROPORTION OF POSITIVE VALUES FROM A NORMAL DISTRIBUTION

C.1 The Problem

C.1.1 Let x be the strength of the fabric (e.g. webbing, cord, etc.) from which a component of a parachute is made. It is tested by loading the fabric until it fails and the force (in pounds) at rupture is the material strength. A random sample of N_x such fabric samples is taken and the material strength of each measured. Let us consider an example in which we have a sample of $N_x = 69$ pieces of fabric with a mean strength $\overline{x}$ of

$$\overline{x} = \frac{\sum_{i=1}^{69} x_i}{69} = 3449 \text{ pounds},$$

and a sample standard deviation s_x of

$$s_x = \sqrt{\frac{\sum_{i=1}^{69} (x_i - \overline{x})^2}{69 - 1}} = \sqrt{32,761} = 181.$$

We assume that these observed material strengths come from a consistent population (or distribution) of material strengths. Let this population have a mean value μ_x and a population standard deviation σ_x. In addition, we assume that the distribution of material strengths is "normal". That is, we assume that:

Probability of a material strength greater than or equal to x is given by

$$\frac{1}{\sqrt{2\pi}\,\sigma_x} \int_x^{\infty} e^{-\frac{(t-\mu_x)^2}{2\sigma_x^2}}\, dt.$$

C.1.2 Similarly, let y be the maximum load stress on the above part in a parachute drop. Test measurements of y are obtained as described in Section 4 of this report. For example, in an actual test, a random sample of $N_y = 11$ such load stresses were obtained and measured, giving a mean value, $\bar{y}$, of

$$\bar{y} = \frac{\sum_{i=1}^{11} y_i}{11} = 1,704 \text{ pounds},$$

and a sample standard deviation of

$$s_y = \sqrt{\frac{\sum_{i=1}^{11} (y_i - \bar{y})^2}{11 - 1}} = \sqrt{147,456} = 384$$

Again, we assume that these observed load stresses come from a consistent population (or distribution) of load stresses. Let this population have a mean value μ_y and a population standard deviation σ_y. In addition, we assume that the distribution of load stresses is "normal".

C.1.3 If we take one random value x of material strength and one random value y of load stress, then this phase of parachute activity will be successful if x is greater than y, and will fail if x is less than y. Let P be the probability of success. Then

$$P = \text{Prob } (x > y) = \text{Prob } (x - y > 0)$$

Under the conditions of (a) and (b), and if we assume that x and y are statistically independent, it is easy to show that the random variable x-y is normally distributed with mean $\mu_x - \mu_y$ and standard deviation $\sqrt{\sigma_x^2 + \sigma_y^2}$. Hence,

$$P = \frac{1}{\sqrt{2\pi (\sigma_x^2 + \sigma_y^2)}} \int_0^{\infty} e^{-\frac{[t - (\mu_x - \mu_y)]^2}{2(\sigma_x^2 + \sigma_y^2)}} dt$$

$$= \int_{\frac{-(\mu_x - \mu_y)}{\sqrt{\sigma_x^2 + \sigma_y^2}}}^{\infty} \frac{e^{-w^2/2}}{\sqrt{2\pi}} dw$$

C.1.4 Sample estimates of μ_x, μ_y, σ_x and σ_y are, respectively, $\bar{x}$, $\bar{y}$, s_x and sy.. The problem is, from these estimates, to

1. Estimate P

2. Describe the variability or quality of this estimate in terms of a confidence interval.

C.1.5 The estimate of P. The estimate of P is obtained by substituting the estimates for the true values. That is, if p is an estimate of P, then

$$p = \int_{-z}^{\infty} \frac{e^{-t^2/2}}{\sqrt{2\pi}}\, dt$$

where we have written

$$z = \frac{\bar{x} - \bar{y}}{s} = \frac{\bar{x} - \bar{y}}{\sqrt{s_1^2 + s_2^2}}$$

In our example

$$\bar{x} - \bar{y} = 3{,}449 - 1{,}704 = 1{,}745$$

$$s = \sqrt{s_1^2 + s_2^2} = 424.52$$

$$z = \frac{\bar{x} - \bar{y}}{s} = 4.11052$$

$$p = \int_{-z}^{\infty} \frac{e^{-t^2/2}}{\sqrt{2\pi}}\, dt = \int_{-4.11052}^{\infty} \frac{e^{-t^2/2}}{\sqrt{2\pi}}\, dt = 0.99998026$$

This is the estimated probability of success. The estimated failure rate is

$$q = 1 - p = 0.00001974 = 2.0 \text{ in } 100{,}000.$$

This statistic is the most convenient estimate of P; a second estimate could be the "uniformly minimum variance unbiased estimate" of P as given in Reference 18. (Because of the transformation from z to p, p is slightly biased.)

In general, both estimates are quite close; our preference at present is for p, which is the easiest to compute.

C.1.6 A confidence interval on P cannot be obtained in this problem because of the existence of the many nuisance parameters. However, an "approximate confidence interval" has been obtained. It is not perhaps a confidence interval, but can be looked on as a figure of merit similar in properties to a confidence interval.

To describe this approximation we shall first describe in Part C.2 the true confidence interval in a problem similar to ours, and then in Part C.3 describe how to obtain an "approximate confidence interval" for our problem based on the results of Part C.2.

C.2 Simplified Problem in Which Confidence Interval is Available

C.2.1 Let us consider the same populations of x, the material strength, and y, the maximum load stress, as in Part C.1.1, but change the sampling procedure as follows.

Take a random observation of x and a random observation of y and let their difference be d = x - y. Assume we do this N times, giving a sample $d_1, d_2, \ldots, d_N$. Now let us estimate P from the d's alone.

Clearly, the expected value of d is

$$E(d) = \mu_d = \mu_x - \mu_y$$

$$\sigma^2(d) = \sigma_d^2 = \sigma_x^2 + \sigma_y^2 .$$

Hence $Z = \dfrac{\mu_x - \mu_y}{\sqrt{\sigma_x^2 + \sigma_y^2}} = \dfrac{\mu_d}{\sigma_d}$

Now if we define the statistics $\bar{d}$ and s_d as

$$\bar{d} = \frac{\sum_{i=1}^{N} d_i}{N}$$

$$s_d = \sqrt{\frac{\sum (d_i - \bar{d})^2}{N - 1}}$$

Then an estimate of z is the statistic z_d where

$$z_d = \frac{\bar{d}}{s_d}$$

and an estimate of P is

$$p_d = \int_{-z_d}^{\infty} \frac{e^{-t^2/2}}{\sqrt{2\pi}}\, dt = \int_{-\frac{\bar{d}}{s_d}}^{\infty} \frac{e^{-t^2/2}}{\sqrt{2\pi}}\, dt$$

C.2.2 Plan: We shall obtain a confidence interval for the population quantity P by first obtaining a confidence interval for the population quantity $Z = \mu_d / \sigma_d$ and then obtain a confidence interval on P by using the fact that P is a monotone increasing function of Z.

C.2.3 Non-Central "t": To keep our notation that of the tables we shall use, 17/ let us define the non-central t statistic as $t = x/\sqrt{w}$ where x is a random variable distributed in a Normal distribution with mean δ and variance 1; w is a random variable distributed as χ^2/f with f degrees of freedom; and x and w are statistically independent. Thus, t has a non-central t distribution with f degrees of freedom and non-central parameter δ.

Let us symbolize the percentage points of t by $t(f,\delta,\epsilon)$ where Prob $[t \geq t(f,\delta,\epsilon)] = \epsilon$. In the Tables of the Non-Central t-Distribution[17] what is tabled is

$$X(f,\delta,\epsilon) = \frac{t(f,\delta,\epsilon)}{\sqrt{f}}$$

For negative values we have the relationship

$$t(f, -\delta, \epsilon) = -t(f, \delta, 1-\epsilon)$$

and hence

$$X(f, -\delta, \epsilon) = -X(f, \delta, 1-\epsilon)$$

C.2.4 Now the statistic

$$\sqrt{N} \cdot z_d = \frac{\sqrt{N}\,\bar{d}}{s_d} = \frac{\sqrt{N}\,\bar{d}/\sigma_d}{s_d/\sigma_d}$$

can be seen to be distributed as non-central t with f = N - 1 degrees of freedom and parameter

$$\delta = \frac{\sqrt{N}\,\mu_d}{\sigma_d} = \frac{\sqrt{N}\,(\mu_x - \mu_y)}{\sqrt{\sigma_x^2 + \sigma_y^2}}$$

Since the numerator $\sqrt{N}\,\bar{d}/\sigma_d$ is distributed normally with mean value $\sqrt{N} \cdot \mu_d/\sigma_d$ and variance 1; and in the denominator s_d^2/σ_d^2 is distributed as $\chi^2/(N-1)$ with N-1 degrees of freedom.

Hence

$$\text{Probability } [t(f,\delta, 1-\beta) \leq \sqrt{N}\, z_d \leq t(f,\delta,\alpha)] = 1-\beta-\alpha$$

$$= \text{Prob}\left[\frac{t(f,\delta, 1-\beta)}{\sqrt{N}} \leq z_d \leq \frac{t(f,\delta,\alpha)}{\sqrt{N}}\right]$$

Now $t(f, \delta, \alpha)$ is, for any fixed f and α, a monotone increasing function of δ. Hence let $\tilde{\delta}_\alpha(z)$ be, for any fixed f and α, the solution of the equation

$$t(f, \tilde{\delta}_\alpha(z), \alpha) = \sqrt{N} \cdot z_d$$

Then the interval

$$z_d \leq \frac{t(f, \delta, \alpha)}{\sqrt{N}}$$

corresponds to the interval

$$\tilde{\delta}_\alpha(z_d) \leq \delta$$

in that all values of the random variable z_d that lie in one interval, lie in the other; and all values of z_d that are outside one interval are outside the other.

Similarly the two intervals

$$\frac{t(f, \delta, 1-\beta)}{\sqrt{N}} \leq z_d$$

and

$$\delta \leq \tilde{\delta}_{1-\beta}(z_d)$$

correspond.

Hence the intervals

$$\frac{t(f, \delta, 1-\beta)}{\sqrt{N}} \leq z_d < \frac{t(f, \delta, \alpha)}{\sqrt{N}}$$

and

$$\tilde{\delta}_\alpha(z_d) < \delta \leq \tilde{\delta}_{1-\beta}(z_d)$$

correspond. Hence

$$\text{Prob}\left[\tilde{\delta}_\alpha(z_d) \leq \delta \leq \tilde{\delta}_{1-\beta}(z_d)\right] = 1-\beta-\alpha$$

which is interpreted meaning that the random interval $\tilde{\delta}_\alpha(z_d)$ to $\tilde{\delta}_{1-\beta}(z_d)$ has a probability of $1-\beta-\alpha$ of containing the true value $\sqrt{N}\,\mu_d/\sigma_d$. Hence, we have a confidence interval of size $1-\beta-\alpha$ for the population parameter $\delta = \sqrt{N}\,\mu_d/\sigma_d$.

C.2.5 In terms of the percentage points tabled in Tables of the Non-Central t-Distribution,[17] we have that $\tilde{\delta}_\alpha$ is the solution of the equation

$$\sqrt{N} \cdot z_d = t\,(f,\ \tilde{\delta}_\alpha(z_d),\alpha) = \sqrt{f} \cdot X\,(f,\ \tilde{\delta}_\alpha(z_d),\alpha)$$

Writing $\tilde{\delta}_\alpha$ for $\tilde{\delta}_\alpha(z_d)$, we have the confidence interval $\tilde{\delta}_\alpha$ to $\tilde{\delta}_{1-\beta}$ given by the equations

$$X\,(f,\tilde{\delta}_\alpha,\alpha) = \sqrt{N/f} \cdot z_d$$

$$X\,(f,\tilde{\delta}_{1-\beta},\ 1-\beta) = \sqrt{N/f} \cdot z_d$$

and this confidence interval is of size $1-\alpha-\beta$.

The tables give the percentage points in terms of

$$K = \frac{\delta}{\sqrt{f+1}} \qquad \text{(instead of in terms of } \delta\text{)}$$

which in our problem is

$$K = \frac{\delta}{\sqrt{f+1}} = \sqrt{\frac{N}{f+1}} \cdot \frac{\mu_d}{\sigma_d}$$

The entry for the percentage points on Page 383 to 389 of the tables is

$$\text{"p"} = \int_K^\infty \frac{e^{-t^2/2}}{\sqrt{2\pi}}\,dt.$$

However, the values of K (called K_p) corresponding to the values of "p" used as an entry, are given on Page 3. Reading the percentage points as a function of K, we solve the equations

$$X\ (f,\ \widetilde{K}_{\alpha},\ \alpha\ ) = \sqrt{N/f} \cdot z_d$$

$$X\ (f,\ \widetilde{K}_{1-\beta}\ ,\ 1-\beta\ ) = \sqrt{N/f} \cdot z_d$$

for $\widetilde{K}\ (\alpha)$ and $\widetilde{K}\ (1-\beta)$ by an inverse interpolation.

Then the interval

$$\sqrt{(f+1)/N}\ \widetilde{K}\ (\alpha)\ \text{to}\ \sqrt{(f+1)/N}\ \widetilde{K}\ (1-\beta)$$

will be the confidence interval of size $1-\beta-\alpha$ on the parameter μ_d/σ_d.

C.2.6 If we wish a one-sided confidence interval, as may seem appropriate, we need only set $\beta = 0$. Then the confidence interval

$$\sqrt{(f+1)/N}\ \ \widetilde{K}\ (\alpha)\ \text{to}\ \infty$$

will be a confidence interval of size $1-\alpha$ on the parameter μ_d/σ_d.

C.2.7 The above discussion has dealt with the population parameter μ_d/σ_d. The population proportion of success is

$$P = \int_{-\frac{\mu_d}{\sigma_d}}^{\infty} \frac{e^{-t^2/2}}{\sqrt{2\pi}}\,dt$$

Since P is a monotone increasing function of μ_d/σ_d, we have only to make the same transformation on the confidence intervals for μ_d/σ_d to obtain confidence intervals for P.

C.2.8 Thus, the interval

$$\int_{-\sqrt{(f+1)/N}\,\tilde{K}(\alpha)}^{\infty} \frac{e^{-t^2/2}}{\sqrt{2\pi}}\,dt \text{ to } \int_{-\sqrt{(f+1)/N}\,\tilde{K}(1-\beta)}^{\infty} \frac{e^{-t^2/2}}{\sqrt{2\pi}}\,dt$$

is the confidence interval of size $1-\beta-\alpha$ on the parameter P.

C.2.9 The interval

$$\int_{-\sqrt{(f+1)/N}\,\tilde{K}(\alpha)}^{\infty} \frac{e^{-t^2/2}}{\sqrt{2\pi}}\,dt \text{ to } 1.$$

is the one-sided confidence interval of size $1-\alpha$ on the parameter α.

C.2.10 Calculation: Interpolation in the percentage points is roughly linear in $\underline{X}$ (f, δ, ϵ) if we consider X to be a function of $K = \delta / \sqrt{f+1}$. The Tables give X (f, δ, ϵ) as a function of

$$"p" = \int_{K}^{\infty} \frac{e^{-t^2/2}}{\sqrt{2\pi}}\,dt = \int_{\delta/\sqrt{f+1}}^{\infty} \frac{e^{-t^2/2}}{\sqrt{2\pi}}\,dt$$

and would seem to imply that interpolation is to be made on "p". This will not be satisfactory; interpolation must be done on K.

A graphic presentation is also possible, plotting

$$K(\alpha) = \sqrt{N/(f+1)} \cdot \mu/\sigma$$

as a function of $\sqrt{N/f}\ \bar{x}_d/s_d$ for each α and f (see Figures 4, 5, and 6).

C.3 Confidence Intervals for the Original Problem

C.3.1 Reducing to the Solution of Part II, An Inefficient Technique

The problem of Part C.1 can be reduced to that of Part C.2 by first noting which sample has the smaller number of observations, second taking a random sample of this size from the larger sample, and third, randomly selecting pairs.

For example, in the example of Part C.1, the y sample was the smaller, with 11 observations. Hence, we can take a random sample of 11 from the 69 observations on x. The resultant sample of 11 x's is then randomly paired with the 11 y's, giving 11 pairs from which we obtain 11 differences x-y. And from here we can proceed as in Part C.2.

This procedure will give a confidence interval which is correct in the sense that probabilities can be exactly determined. Since we are essentially throwing away data, however, it will be inefficient. That is, for any given probability of coverage of the true parameter (usually called the "size" of the confidence interval), this confidence interval will be wider than that computed on the basis of all the observation.

C.3.2 The distribution of

$$z = \frac{\bar{x} - \bar{y}}{\sqrt{s_x^2 + s_y^2}}$$

depends not only on the parameter

$$Z = \frac{\mu_x - \mu_y}{\sqrt{\sigma_x^2 + \sigma_y^2}}$$

but also involves nuisance parameters. Hence a confidence interval in the classical sense cannot be obtained. However, a quite satisfactory approximation has been obtained.

In this Part of the discussion we shall give a rather straight forward derivation of the approximation. In Part C.4 we shall examine more deeply to try to see how good the approximation is, or rather, why it is a good approximation.

C.3.3 The plan is as follows: we shall examine the means and variances of the numerator and denominator of the statistic

$$\frac{\bar{d}}{s_d}$$

that occurs in Part C.2 and those of the statistic

$$\frac{\bar{x} - \bar{y}}{\sqrt{s_x^2 + s_y^2}}$$

that occurs in Part C.1, and try to equate them in some way.

For convenience, we shall consider the square of the denominators. (The results are the same.)

Examination shows

$$E(\bar{d}) = \mu_x - \mu_y \quad ; \quad E(\bar{x}-\bar{y}) = \mu_x - \mu_y$$

$$\sigma^2(\bar{d}) = \frac{\sigma x^2 + \sigma y^2}{N} = \frac{\sigma d^2}{N} \quad ; \quad \sigma^2(\bar{x}-\bar{y}) = \frac{\sigma x^2}{N_x} + \frac{\sigma y^2}{N_y}$$

$$E(s_d^2) = \sigma_x^2 + \sigma_y^2 = \sigma_d^2 \quad ; \quad E(s_x^2 + s_y^2) = \sigma_x^2 + \sigma_y^2 = \sigma_d^2$$

$$\sigma^2(s_d^2) = \frac{2\sigma d^4}{f} \quad ; \quad \sigma^2(s_x^2 + s_y^2) = \frac{2\sigma x^4}{f_x} + \frac{2\sigma y^4}{f_y}$$

The variance of the sample variance is an exact result for samples from a normal distribution.19/

We note that the mean values are the same. We also see that, in the problem of Part C.1, if we define

$$N = \frac{\sigma x^2 + \sigma y^2}{\dfrac{\sigma x^2}{N_x} + \dfrac{\sigma y^2}{N_y}}$$

and

$$f = \frac{(\sigma x^2 + \sigma y^2)^2}{\frac{\sigma x^4}{f_x} + \frac{\sigma y^4}{f_y}}$$

Then

$$\sigma^2 \ (\bar{x}-\bar{y}) = \frac{\sigma x^2 + \sigma y^2}{N}$$

and

$$\sigma^2 \ (s_x^2 + s_y^2) = \frac{2(\sigma x^2 + \sigma y^2)^2}{f}$$

The variance of the numerator and the variance of the denominator are now in the same form as those for Part C.2. Hence if the above non-integer values of N and f are used in the solution of Part C.2, we will have an approximate solution to the problem of Part C.1.

Unfortunately, we cannot compute N and f this way because we do not know σ_x and σ_y. Hence the next approximation is to substitute s_x and s_y for σ_x and σ_y, obtaining

$$\tilde{N} = \frac{s_x^2 + s_y^2}{\frac{s_x^2}{N_x} + \frac{s_y^2}{N_y}}$$

$$\tilde{f} = \frac{(s_x^2 + s_y^2)^2}{\frac{s_x^4}{f_x} + \frac{s_y^4}{f_y}}$$

C.3.4 The recommended approximate procedure is as follows: Compute $\tilde{N}$ and $\tilde{f}$; substitute these numerical values for N and f in the equations of Part C.2 and thereby compute the approximate confidence interval. Experience has shown that the above equations are quite cumbersome for computation purposes. Considerable ease of calculation can be gained by the following technique:

Let

s_1^2 = smaller of s_x^2 or s_y^2

s_2^2 = larger of s_x^2 or s_y^2

N_1 = sample size corresponding to s_1^2

N_2 = sample size corresponding to s_2^2

$f_1 = N_1-1$ = degrees of freedom corresponding to s_1^2

$f_2 = N_2-1$ = degrees of freedom corresponding to s_2^2

Then let

$$r = \frac{s_1^2}{s_2^2}$$

$$\theta = \frac{N_2}{N_1}$$

$$g = \frac{f_2}{f_1}$$

Then

$$\tilde{N} = N_2 \frac{(1+r)}{1 + \theta r} = N_1 \frac{(1+r)}{\frac{1}{\theta}+r}$$

$$\tilde{f} = f_2 \frac{(1+r)^2}{1+gr^2} = f_1 \frac{(1+r)^2}{\frac{1}{g}+r^2}$$

If θ (and hence g) is less than 1, we use the first equations in N_2 and f_2. If θ (and hence g) is greater than 1, we use the second equations in N_1 and f_2, first computing

$$\frac{1}{\theta} = \frac{N_1}{N_2}$$

$$\frac{1}{g} = \frac{f_1}{f_2}$$

C.3.5 In the example of Part C.1 we have

$$r = \frac{s_1^2}{s_2^2} = \frac{s_x^2}{s_y^2} = \frac{147,456}{32,761} = 0.222175$$

$$\theta = \frac{N_2}{N_1} = \frac{N_y}{N_x} = \frac{11}{69} = 0.159420$$

$$g = \frac{f_2}{f_1} = \frac{f_y}{f_x} = \frac{10}{68} = 0.147059$$

Hence

$$\widetilde{N} = \frac{s_x^2 + s_y^2}{\dfrac{s_x^2}{N_x} + \dfrac{s_y^2}{N_y}} = \frac{32,761 + 147,456}{\dfrac{32,761}{69} + \dfrac{147,456}{11}} = 12.9840$$

$$= N_2 \frac{(1 + r)}{1 + \theta r} = 11 \frac{(1.222175)}{(1.03542)} = 12.9840$$

$$\widetilde{f} = \frac{(s_x^2 + s_y^2)^2}{\dfrac{s_x^4}{f_x} + \dfrac{s_y^4}{f_y}} = \frac{(180,217)^2}{\dfrac{(32,761)^2}{68} + \dfrac{(147,456)^2}{10}} = 14.8295$$

$$= f_2 \frac{(1+r)^2}{1+gr^2} = \frac{10 \ (1.222175)^2}{1.007260} = 14.8295$$

Hence we have

$$\sqrt{\widetilde{N}/\widetilde{f}} \ \ z = \sqrt{\widetilde{N}/\widetilde{f}} \ \frac{(\bar{x} - \bar{y})}{\sqrt{s_x^2 + s_y^2}} = \sqrt{\frac{12.9840}{14.8295}} = 3.84627$$

Looking into the Tables of the Non-Central t-Distribution[17/] for $\epsilon = 0.10$ and $f = 14$, 15, and 16, we find

		X (f, K, 0.10)			
		f = degrees of freedom			
P	K	14	15	16	14.8295
.0100	2.3263 48	3.325 (49)	3.276 (43)	3.233	
.0040	2.6520 70	3.766 (54)	3.712 (48)	3.664	3.7212
.0025	2.8070 34	3.977 (58)	3.919 (50)	3.869	3.9289
.0010	3.0902 32	4.363 (62)	4.301 (56)	4.245	

The terms in parentheses have been supplied -- they are the first differences. Looking at them we see that interpolation for degrees of freedom is linear. That is, the effect of second difference terms (meaning fitting a second degree polynomial) is at most equal to 0.0005.

Similarly, for each of the three values of f, we find that interpolation of X vs. K is linear. Here, checking is more difficult as the K's are not tabulated at equal intervals. However, second and third degree polynomial interpolation can be done by the use of divided differences or by the use of Lagrangian Interpolation Formulas. Checking in this manner, we find that interpolation for K is linear; higher order interpolation gives negligible adjustments.

Hence, interpolating for f = 14.8295, we find the appropriate values of X(14.8295, K, 0.10) given above. Interpolating K-wise to solve:

$$X(14.8295,\ K,\ 0.10) = \sqrt{\tilde{N}/\tilde{f}}\ .\ \frac{(\bar{x} - \bar{y})}{\sqrt{s_1^2 + s_2^2}} = 3.846\ 27$$

we have

$$\frac{K - 2.6520\ 70}{2.8070\ 34 - 2.6520\ 70} = \frac{3.846\ 27 - 3.7212}{3.9289 - 3.7212} = \frac{.12507}{.2077}$$

hence

$$K = 2.652070 + \left(\frac{0.12507}{0.2077}\right)\ 0.1549\ 64 = 2.745\ 384$$

Hence the 90% lower confidence value is

$$\left(\frac{\mu}{\sigma}\right)_{.90} = \sqrt{\frac{\tilde{f}+1}{\tilde{N}}} \cdot K = \sqrt{\frac{15.8295}{12.9840}} \cdot K = \sqrt{1.2191\ 47} \cdot K = 3.0313\ 2$$

Hence the 90% lower confidence value of P is

$$P_{.90} = \int_{-3.03132}^{\infty} \frac{e^{-t^2/2}}{\sqrt{2\pi}}\, dt = .998783$$

The failure rate is $Q_{.90} = 1 - P_{.90} = 0.001217 \approx 1.2$ in 1000.

To summarize; in this example, our estimate of P is 0.9999803 with a 90% confidence interval of .99878 to 1. (That is, this interval has a 90% chance of including the true value of P. The lower bound, .99878, is the value used as the reliability at a 90% confidence coefficient.)

Alternatively, the estimate of the failure rate Q = 1 - P is .0000197 with a 90% confidence interval of 0. to 0.00123.

C.3.6 Note: If we look at the differences f-wise (which are given in parentheses) we note a possible error. The second differences, e.g., the differences between the quantities in parentheses are, respectively, 6, 6, 8 and 6 for p = .010, .004, .0025 and .001. Clearly something is wrong with the values for p = .0025. If we take differences K-wise, we see that the trouble lies in the value for 16 Degrees of Freedom. If this is changed from 3.869 to 3.867, the second differences will be satisfactory. However, this change would not affect the linear interpolation.

C.3.7 A speed-up in calculation is obtained by using a graphical procedure. For any fixed ϵ and f, we can plot the line of K vs. X (f, K, ϵ) (see Figures 4, 5 and 6). Entering with

$$X = \sqrt{\tilde{N}/\tilde{f}} \cdot \frac{(\bar{x} - \bar{y})}{\sqrt{s_1^2 + s_2^2}}$$

we read K, for the appropriate ϵ and f, and obtain the appropriate confidence interval as

$$\frac{\mu}{\sigma} = \sqrt{\frac{\tilde{f}+1}{\tilde{N}}} \cdot K$$

C.4 Examination and Justification of Approximation

C.4.1 The Problem. The argument by which the approximation is developed in Part C.3 is not the only way the approximation can be developed. Another argument, which in many ways clarifies aspects not considered in the argument of Part C.3, is as follows.

We wish to obtain a confidence interval estimate of the population parameter P, where

$$P = \int_{-\frac{(\mu_x - \mu_y)}{\sqrt{s_x^2 + s_y^2}}}^{\infty} \frac{e^{-t^2/2}}{\sqrt{2\pi}} dt = \int_{-Z}^{\infty} \frac{e^{-t^2/2}}{\sqrt{2\pi}} dt$$

There are four unknown parameters μ_x, μ_y, σ_x and σ_y (we assume N_1, N_2, f_1, f_2 are known), and a confidence interval estimate of one function of them (in this case, P) cannot be obtained in any known method.

C.4.2 The First Approximation Step. Clearly the sufficient statistics are $\bar{x}$, $\bar{y}$, s_x and s_y. While the upper and lower confidence intervals are functions of these four sufficient statistics, we do not know what the best function is. However, we shall assume that a satisfactory confidence interval can be obtained on the basis of the distribution of the statistics

$$z = \frac{\bar{x} - \bar{y}}{\sqrt{s_x^2 + s_y^2}}$$

in a manner analogous to the development of Part C.2, assuming that any possible inefficiency thereby introduced will be small. This is the First Approximation Step.

C.4.3 Reduction to One Nuisance Parameter. The nuisance parameters must next be considered. It is easy to show that the distribution of the statistic z only depends on the parameters.

$$Z = \frac{\mu_x - \mu_y}{\sqrt{\sigma_x^2 + \sigma_y^2}}$$

and

$$R = \frac{\sigma_x^2}{\sigma_y^2}$$

Hence, we now have only one nuisance parameter, R. The problem is now to obtain a confidence interval estimate of P for given known values of the variance ratio R.

C.4.4 The Second Approximation Step and Its Implications. The statistic z is not distributed as a constant times non-central t, because $s_x^2 + s_y^2$ is not distributed as a constant times χ^2. A basic approach would be to develop the exact distribution of z, but this is a very involved process. However, the distribution of $s_x^2 + s_y^2$ is quite satisfactorily approximated by a constant times χ^2. Using the technique of equating the first and second moments of $s_x^2 + s_y^2$ and $z\chi^2$, as described in Part C.3, we have

$$\frac{s_x^2 + s_y^2}{\sigma_x^2 + \sigma_y^2}$$

approximately distributed as $\frac{\chi^2}{f}$ with f degrees of freedom where

$$f = \frac{(\sigma_x^2 + \sigma_y^2)^2}{\frac{\sigma_x^4}{f_x} + \frac{\sigma_y^4}{f_y}} = \frac{(R+1)^2}{\frac{R^2}{f_x} + \frac{1}{f_y}}$$

Note that f only depends on R; Z does not enter. This is the Second Approximation Step.

Dividing numerator and denominator of the statistic z by $\sqrt{\sigma_1^2 + \sigma_2^2}$, we obtain

$$z = \frac{(\bar{x} - \bar{y}) / \sqrt{\sigma_x^2 + \sigma_y^2}}{\sqrt{(s_x^2 + s_y^2) / (\sigma_x^2 + \sigma_y^2)}}$$

The denominator is approximately distributed as χ^2/f and hence we now have the appropriate denominator statistic for non-central t. The numerator is normally distributed with mean $\mu_x - \mu_y$ and variance

$$\frac{\dfrac{\sigma_x^2}{N_x} + \dfrac{\sigma_y^2}{N_y}}{\sigma_x^2 + \sigma_y^2}$$

Hence z is approximately distributed as a constant times non-central t, with f degrees of freedom, and

$$z \sqrt{\frac{\sigma_x^2 + \sigma_y^2}{\dfrac{\sigma_x^2}{N_x} + \dfrac{\sigma_y^2}{N_y}}}$$

is approximated distributed as non-central t with f degrees of freedom and non-central parameter

$$\delta = \frac{\mu_x - \mu_y}{\sqrt{\sigma_x^2 + \sigma_y^2}} \sqrt{\frac{\sigma_x^2 + \sigma_y^2}{\dfrac{\sigma_x^2}{N_x} + \dfrac{\sigma_y^2}{N_y}}}$$

Now if we define N as

$$N = \frac{\sigma_x^2 + \sigma_y^2}{\dfrac{\sigma_x^2}{N_x} + \dfrac{\sigma_y^2}{N_y}} = \frac{R + 1}{\dfrac{R}{N_x} + \dfrac{1}{N_y}}$$

(N only depends on R) we have that the statistic $z\sqrt{N}$ is approximately distributed as non-central t with f degrees of freedom and non-central parameter

$$\delta = \frac{(\mu_x + \mu_y)\sqrt{N}}{\sqrt{\sigma_x^2 + \sigma_y^2}}$$

We see that, to this point, while the introduction of the equation for f (the degrees of freedom) did involve an approximation (the second approximation step), the introduction of N was purely a symbolic convenience. It did not introduce another approximation.

C.4.5 Recapitulation. Thus, if we know the true population variance ratio

$$R = \sigma_x^2 / \sigma_y^2$$

we can compute f and N as given above. Substitution of these values into the technique described in Part C.2 will give an excellent approximate confidence interval.

C.4.6 The Third Approximation Step. However, R, the true population variance ratio is not known, and it is necessary to resort to a Third Approximation Step. We replace R by $\tilde{R}$, the sample variance ratio,

$$\tilde{R} = \frac{s_x^2}{s_y^2}$$

in the equations for f and N. Call the results $\tilde{f}$ and $\tilde{N}$. That is

$$\tilde{f} = \frac{(\tilde{R} + 1)^2}{\frac{\tilde{R}^2}{f_x} + \frac{1}{f_y}} = \frac{(s_x^2 + s_y^2)^2}{\frac{s_x^4}{f_x} + \frac{s_y^4}{f_y}}$$

$$\tilde{N} = \frac{\tilde{R} + 1}{\frac{\tilde{R}}{N_x} + \frac{1}{N_y}} = \frac{s_x^2 + s_y^2}{\frac{s_x^2}{N_x} + \frac{s_y^2}{N_y}}$$

Using these approximations to f and N, we have obtained the confidence intervals given in Part C.3.3.

C.4.7 Additional Comments. These approximate confidence intervals appear to be entirely satisfactory for this application. One possible check is to note the effect of a change in R on the answer. If it is negligible, then clearly the third approximation step is quite good (the first two have been shown to be very good). If we try doubling $\tilde{R}$ or halving $\tilde{K}$ in the example given, the effect on the confidence interval is quite small, so we can be quite confident in the confidence interval obtained.

However, this check does not work both ways; even if the effect of doubling $\tilde{R}$ or halving it is large, the approximate confidence intervals may be quite correct.

C.5 Assumptions of Statistical Independence and Normality

To summarize the content of this section, one must consider the estimate, p, of successful operation and the confidence interval about it, given in Parts C.1 to C.4, as figures of merit. It appears that they are quite satisfactory figures of merit.

C.5.1 Reviewing the previous paragraphs, we see that in addition to the three approximation steps described above in part C.4, we have made two other assumptions: statistical independence and Normality. We have assumed that the distributions of x, the material strength and y, the applied load stress, are statistically independent. This seems to be quite appropriate within the accuracy of the kind of data we are using. It is hard to see how x and y can be correlated; about the only possibility is that certain atmospheric conditions affect both x and y, but even here it is probable that the distribution of x-y is not affected.

C.5.2 A more disturbing assumption is that the difference x-y is Normally Distributed. That is, we assume that the probability that x-y is larger than W is

$$\int_W^{\infty} \frac{e^{-(t-\mu_d)^2/2\sigma_d^2}}{\sqrt{2\pi}\ \sigma_d}\, dt$$

This means that if we can approximate the two constants μ_d and σ_d, we can compute any probability.

In many statistical problems, careful examination has shown that the answers are quite insensitive to Normality, so that the assumption of an underlying Normal Distribution may be warranted since, even marked deviations from Normality will only slightly affect the results. However, this occurs only near the center of the distribution; it surely does not apply when we are trying to estimate proportions far out in the tails. A distribution can be satisfactory approximated by a Normal Distribution for the central 90%, but may differ quite markedly beyond the 95% point.

It is quite dubious whether we can estimate the 2 in 100,000 point by samples of 69 and 11 observations which seem to be equivalent to less than 18 pairs of measurements of x-y. To project to the 0.00002 point, we use the Normal Distribution. But to check the correctness of this we clearly need at least 100,000 observation pairs.

Thus, it is very clear that the estimate of failure rate is very dependent on the assumption of Normality. However, it will not be possible to obtain 100,000 observation pairs, so our answer is a good current guess and is surely an excellent figure of merit, as is the corresponding confidence interval.

C.5.3 However, the approximations made can be shown to be fairly good. If the method for obtaining the estimate p is examined, we note that there was an intermediate step in which μ_d/σ_d was estimated from

$$z = \frac{\bar{x} - \bar{y}}{\sqrt{s_x^2 + s_y^2}}.$$

Now z, the estimated of μ_d/σ_d, is a typical statistic of the kind described at the beginning of this section. It is very robust, that is its distribution is quite insensitive to Normality; or, to put it another way, its distribution is insensitive to the true underlying population. (Even this small effect is easily taken into account; it is primarily dependent on the deviation of the fourth central moment from $3\sigma^4$). The confidence interval on z is more dependent on the underlying population, but is still fairly insensitive.

The sensitivity to Normality arises in converting to the corresponding proportion by the transformations

$$p = \int_{-\frac{\mu_d}{\sigma_d}}^{\infty} \frac{e^{-t^2/2}}{\sqrt{2\pi}}\, dt$$

$$p = \int_{-z}^{\infty} \frac{e^{-t^2/2}}{\sqrt{2\pi}}\, dt$$

Here, the Normal assumption is directly involved. If we know the shape of the underlying population and it was not Normal, we would use another transform, not the above Normal one. That is, if we had enough data to determine the shape of the underlying population (say 100,000 pairs of observations), then we could obtain a much better estimate of P by calculating a more appropriate transformation from z or perhaps even from $\overline{x}-\overline{y}$ and $\sqrt{s_x^2 + s_y^2}$.

Thus, if we wish to estimate P, the true proportion of successes, the error due to the Normality assumption may be (and presumably is) much larger than the errors due to all the other assumptions and approximations.. Hence our conclusion that the estimate p and its corresponding confidence interval can only be looked on as appropriate figures of merit for evaluation or comparison. The estimate p is the figure of merit relating to performance; the confidence interval gives some sense of the effect of sampling errors on this figure of merit.

C.5.4 It may also be seen that since the estimate z and the confidence interval about it are robust (that is, insensitive to deviations from Normality in the underlying population), they could be used as figures of merit used. Consideration should be given to this concept in future work.

There is an intermediate position possible which states that since p and z are mathematically related by

$$p = \int_{-z}^{\infty} \frac{e^{-t^2/2}}{\sqrt{2\pi}}\, dt$$

we have that z is a monotone function of p (and vice versa), so that if only p is given, we can still compute z. Hence, the use of p computed this way as a figure of merit is the same as using z as a figure of merit. Therefore, we can look on the use of p, computed as above from z when we do not know the true underlying population, as really being equivalent to the use of z as a figure of merit.

C.6 Tables of Percentage Points of the Non-Central t-Distribution

The following tables of the Percentage Points of the Non-Central t-Statistic are reproduced from pages 383 through 389 of Reference 17.*/ To develop the plots presented in Figures 4, 5, and 6 of Section 4, the confidence coefficient has been taken as $1.0 - \epsilon$. The conversion from the p of the tables to the K of Figures 4, 5, and 6 and equation 14 of Section 4.3.1 (see paragraph C.2.10 of this appendix) is given on page 3 of Reference 17:

p	K
.2500	0.674490
.1500	1.036433
.1000	1.281552
.0650	1.514102
.0400	1.750686
.0250	1.959964
.0100	2.326348
.0400	2.652070
.0025	2.807034
.0010	3.090232

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PERCENTAGE POINTS OF t, THE NON-CENTRAL t-STATISTIC. THE ENTRIES IN THE TABLE GIVE THE VALUES OF x SUCH THAT $Pr\ [t/\sqrt{f} > x] = \epsilon$

f is the number of degrees of freedom; the non-centrality parameter is $\sqrt{f+1}\,K_p$.
K_p is the standardized normal deviate exceeded with probability p.

DEGREES OF FREEDOM 2

p \ ε	.995	.99	.95	.90	.75	.50	.25	.10	.05
.2500	-2.175	-1.474	-.428	-.092	.378	.947	1.793	3.187	4.659
.1500	-.874	-.515	.116	.378	.827	1.474	2.538	4.371	6.337
.1000	-.328	-.087	.410	.655	1.117	1.835	3.063	5.215	7.542
.0650	.033	.216	.655	.898	1.386	2.180	3.571	6.037	8.708
.0400	.319	.469	.882	1.132	1.654	2.531	4.095	6.885	9.921
.0250	.521	.661	1.072	1.331	1.888	2.842	4.562	7.646	11.006
.0100	.823	.956	1.384	1.667	2.291	3.386	5.384	8.990	12.924
.0040	1.057	1.196	1.648	1.956	2.645	3.868	6.118	10.193	14.644
.0025	1.161	1.305	1.771	2.091	2.812	4.098	6.469	10.768	15.466
.0010	1.350	1.498	1.992	2.336	3.116	4.517	7.110	11.820	16.971

DEGREES OF FREEDOM 3

p \ ε	.995	.99	.95	.90	.75	.50	.25	.10	.05
.2500	-1.080	-.774	-.194	.042	.408	.853	1.450	2.277	3.024
.1500	-.356	-.168	.253	.458	.820	1.320	2.046	3.099	4.064
.1000	-.008	.142	.513	.712	1.089	1.638	2.464	3.680	4.807
.0650	.257	.387	.738	.941	1.340	1.942	2.867	4.247	5.529
.0400	.487	.606	.953	1.164	1.590	2.251	3.281	4.831	6.277
.0250	.667	.783	1.134	1.355	1.810	2.524	3.651	5.354	6.945
.0100	.947	1.067	1.439	1.680	2.189	3.003	4.302	6.278	8.131
.0040	1.179	1.302	1.700	1.963	2.522	3.429	4.884	7.108	9.194
.0025	1.284	1.410	1.822	2.095	2.680	3.631	5.162	7.506	9.707
.0010	1.469	1.602	2.041	2.335	2.967	4.000	5.670	8.231	10.641

DEGREES OF FREEDOM 4

p \ ε	.995	.99	.95	.90	.75	.50	.25	.10	.05
.2500	-.700	-.500	-.074	.118	.429	.807	1.288	1.898	2.403
.1500	-.142	-.004	.335	.510	.824	1.246	1.821	2.583	3.226
.1000	.146	.266	.580	.755	1.083	1.545	2.193	3.066	3.809
.0650	.381	.490	.797	.977	1.325	1.829	2.551	3.534	4.376
.0400	.592	.698	1.007	1.195	1.568	2.119	2.919	4.018	4.961
.0250	.766	.869	1.186	1.383	1.780	2.375	3.247	4.451	5.487
.0100	1.038	1.148	1.488	1.704	2.148	2.824	3.825	5.217	6.419
.0040	1.269	1.385	1.749	1.984	2.472	3.223	4.342	5.904	7.254
.0025	1.374	1.492	1.870	2.116	2.626	3.412	4.588	6.231	7.655
.0010	1.564	1.688	2.092	2.355	2.906	3.759	5.039	6.833	8.386

DEGREES OF FREEDOM 5

p \ ε	.995	.99	.95	.90	.75	.50	.25	.10	.05	.01	.005
.2500	-.495	-.346	.003	.170	.445	.780	1.191	1.687	2.076	3.121	3.650
.1500	-.018	.097	.392	.549	.830	1.203	1.690	2.300	2.790	4.119	4.800
.1000	.247	.348	.630	.788	1.084	1.490	2.037	2.732	3.293	4.829	5.625
.0650	.467	.564	.843	1.007	1.321	1.764	2.370	3.149	3.782	5.525	6.418
.0400	.669	.765	1.050	1.222	1.559	2.042	2.712	3.580	4.289	6.250	7.250
.0250	.838	.934	1.228	1.409	1.768	2.288	3.017	3.966	4.743	6.887	8.000
.0100	1.111	1.213	1.530	1.729	2.130	2.720	3.553	4.647	5.545	8.034	9.350
.0040	1.341	1.450	1.791	2.008	2.449	3.103	4.033	5.258	6.265	9.067	10.525
.0025	1.450	1.561	1.914	2.140	2.600	3.286	4.262	5.550	6.610	9.550	11.100
.0010	1.640	1.760	2.136	2.379	2.876	3.619	4.681	6.086	7.243	10.450	12.150

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PERCENTAGE POINTS OF t, THE NON-CENTRAL t-STATISTIC. THE ENTRIES IN THE TABLE GIVE THE VALUES OF x SUCH THAT $Pr\ [t/\sqrt{f} > x] = \epsilon$.

f is the number of degrees of freedom; the non-centrality parameter is $\sqrt{f+1}\ K_p$.
K_p is the standardized normal deviate exceeded with probability p.

DEGREES OF FREEDOM 6

p \ ε	.995	.99	.95	.90	.75	.50	.25	.10	.05	.01	.005
.2500	-.370	-.245	.059	.208	.459	.782	1.127	1.550	1.872	2.489	3.087
.1500	.069	.170	.435	.579	.837	1.174	1.603	2.120	2.520	3.577	4.066
.1000	.320	.411	.668	.815	1.087	1.454	1.994	2.520	2.976	4.165	4.787
.0650	.530	.620	.879	1.031	1.321	1.721	2.251	2.906	3.419	4.787	5.430
.0400	.729	.820	1.086	1.246	1.556	1.992	2.576	3.303	3.877	5.388	6.150
.0250	.895	.987	1.263	1.432	1.763	2.232	2.866	3.660	4.287	5.940	6.775
.0100	1.169	1.287	1.565	1.752	2.121	2.652	3.377	4.290	5.013	6.925	7.868
.0040	1.403	1.505	1.828	2.031	2.437	3.025	3.833	4.855	5.665	7.813	8.900
.0025	1.511	1.617	1.952	2.163	2.587	3.203	4.050	5.124	5.977	8.233	9.375
.0010	1.703	1.818	2.175	2.403	2.860	3.527	4.448	5.618	6.527	9.017	10.250

DEGREES OF FREEDOM 7

p \ ε	.995	.99	.95	.90	.75	.50	.25	.10	.05	.01	.005
.2500	-.280	-.170	.101	.238	.470	.749	1.080	1.454	1.729	2.408	2.730
.1500	.134	.224	.470	.603	.844	1.154	1.541	1.994	2.335	3.128	3.600
.1000	.376	.460	.700	.837	1.091	1.429	1.860	2.372	2.760	3.737	4.213
.0650	.583	.666	.910	1.053	1.323	1.690	2.160	2.737	3.173	4.275	4.800
.0400	.779	.864	1.115	1.266	1.556	1.956	2.480	3.114	3.599	4.834	5.434
.0250	.945	1.030	1.293	1.452	1.761	2.192	2.760	3.449	3.980	5.330	5.983
.0100	1.218	1.312	1.596	1.772	2.117	2.604	3.252	4.044	4.654	6.212	6.975
.0040	1.453	1.553	1.860	2.052	2.432	2.970	3.691	4.576	5.260	7.011	7.850
.0025	1.561	1.665	1.984	2.184	2.580	3.144	3.901	4.830	5.550	7.388	8.268
.0010	1.759	1.868	2.210	2.425	2.852	3.463	4.285	5.297	6.079	8.088	9.050

DEGREES OF FREEDOM 8

p \ ε	.995	.99	.95	.90	.75	.50	.25	.10	.05	.01	.005
.2500	-.212	-.113	.135	.263	.479	.740	1.044	1.381	1.625	2.211	2.483
.1500	.186	.270	.498	.624	.850	1.139	1.494	1.900	2.199	2.930	3.269
.1000	.421	.501	.727	.857	1.095	1.410	1.805	2.262	2.603	3.439	3.830
.0650	.626	.705	.935	1.071	1.326	1.668	2.102	2.612	2.993	3.930	4.373
.0400	.820	.901	1.141	1.284	1.558	1.930	2.408	2.972	3.396	4.443	4.937
.0250	.985	1.069	1.319	1.470	1.762	2.162	2.680	3.294	3.757	4.900	5.450
.0100	1.262	1.350	1.623	1.791	2.116	2.568	3.158	3.862	4.394	5.716	6.350
.0040	1.497	1.593	1.889	2.071	2.429	2.929	3.585	4.372	4.967	6.450	7.150
.0025	1.608	1.706	2.013	2.204	2.578	3.101	3.789	4.615	5.241	6.800	7.533
.0010	1.806	1.912	2.240	2.445	2.848	3.415	4.162	5.061	5.742	7.440	8.250

DEGREES OF FREEDOM 9

p \ ε	.995	.99	.95	.90	.75	.50	.25	.10	.05	.01	.005
.2500	-.158	-.068	.164	.283	.487	.732	1.016	1.325	1.544	2.060	2.293
.1500	.229	.307	.522	.642	.855	1.128	1.457	1.827	2.095	2.737	3.030
.1000	.459	.535	.750	.873	1.100	1.396	1.761	2.177	2.482	3.213	3.550
.0650	.663	.737	.958	1.087	1.329	1.650	2.053	2.515	2.856	3.678	4.050
.0400	.856	.934	1.164	1.301	1.560	1.910	2.351	2.863	3.241	4.156	4.572
.0250	1.020	1.101	1.342	1.486	1.763	2.139	2.617	3.174	3.586	4.587	5.050
.0100	1.298	1.385	1.647	1.807	2.116	2.541	3.085	3.723	4.196	5.350	5.883
.0040	1.536	1.630	1.914	2.089	2.429	2.898	3.503	4.214	4.744	6.034	6.634
.0025	1.647	1.744	2.039	2.222	2.577	3.068	3.702	4.448	5.005	6.360	7.000
.0010	1.847	1.950	2.267	2.464	2.846	3.378	4.067	4.879	5.484	6.960	7.650

PERCENTAGE POINTS OF t, THE NON-CENTRAL t-STATISTIC. THE ENTRIES IN THE TABLE GIVE THE VALUES OF x SUCH THAT $Pr\ [t/\sqrt{f} > x] = \epsilon$

f is the number of degrees of freedom; the non-centrality parameter is $\sqrt{f+1}\,K_p$.

K_p is the standardized normal deviate exceeded with probability p

DEGREES OF FREEDOM 10

p \ ε	.995	.99	.95	.90	.75	.50	.25	.10	.05	.01	.005
.2500	-.114	-.088	.187	.301	.494	.727	.993	1.279	1.480	1.944	2.150
.1500	.265	.338	.543	.657	.861	1.118	1.427	1.768	2.013	2.585	2.842
.1000	.492	.564	.770	.888	1.104	1.384	1.726	2.110	2.386	3.038	3.332
.0650	.693	.766	.978	1.102	1.332	1.636	2.012	2.438	2.747	3.480	3.809
.0400	.888	.962	1.183	1.315	1.563	1.893	2.306	2.777	3.119	3.935	4.300
.0250	1.052	1.130	1.362	1.501	1.765	2.121	2.567	3.079	3.452	4.345	4.750
.0100	1.332	1.415	1.668	1.822	2.118	2.519	3.026	3.612	4.040	5.064	5.534
.0040	1.571	1.661	1.936	2.105	2.429	2.873	3.437	4.089	4.568	5.714	6.237
.0025	1.684	1.777	2.063	2.238	2.577	3.041	3.632	4.316	4.820	6.025	6.583
.0010	1.886	1.985	2.292	2.481	2.846	3.349	3.990	4.735	5.282	6.591	7.200

DEGREES OF FREEDOM 11

p \ ε	.995	.99	.95	.90	.75	.50	.25	.10	.05	.01	.005
.2500	-.077	.004	.208	.316	.500	.722	.974	1.241	1.427	1.850	2.034
.1500	.296	.365	.561	.670	.865	1.111	1.402	1.720	1.945	2.464	2.694
.1000	.520	.590	.788	.901	1.108	1.375	1.696	2.053	2.308	2.896	3.158
.0650	.721	.791	.995	1.114	1.336	1.625	1.979	2.375	2.659	3.320	3.619
.0400	.914	.988	1.201	1.328	1.566	1.880	2.268	2.705	3.020	3.759	4.090
.0250	1.079	1.156	1.380	1.514	1.768	2.106	2.525	3.000	3.343	4.145	4.509
.0100	1.361	1.441	1.688	1.836	2.120	2.501	2.978	3.521	3.914	4.839	5.250
.0040	1.602	1.689	1.957	2.119	2.431	2.852	3.382	3.986	4.425	5.462	5.923
.0025	1.715	1.806	2.084	2.253	2.578	3.019	3.574	4.209	4.670	5.757	6.250
.0010	1.919	2.017	2.314	2.496	2.847	3.325	3.927	4.617	5.118	6.300	6.833

DEGREES OF FREEDOM 12

p \ ε	.995	.99	.95	.90	.75	.50	.25	.10	.05	.01	.005
.2500	-.044	.031	.226	.329	.506	.718	.957	1.209	1.383	1.772	1.940
.1500	.322	.389	.577	.682	.870	1.105	1.381	1.680	1.890	2.368	2.575
.1000	.545	.612	.803	.913	1.112	1.367	1.672	2.007	2.243	2.786	3.027
.0650	.745	.814	1.011	1.126	1.339	1.616	1.951	2.322	2.585	3.192	3.458
.0400	.940	1.010	1.217	1.340	1.569	1.869	2.237	2.646	2.937	3.613	3.910
.0250	1.106	1.178	1.397	1.526	1.770	2.094	2.491	2.935	3.253	3.991	4.320
.0100	1.387	1.466	1.705	1.849	2.122	2.486	2.937	3.445	3.808	4.655	5.029
.0040	1.631	1.715	1.975	2.132	2.433	2.835	3.336	3.901	4.307	5.250	5.675
.0025	1.745	1.833	2.103	2.267	2.580	3.001	3.526	4.119	4.545	5.537	5.988
.0010	1.950	2.044	2.335	2.511	2.849	3.305	3.874	4.518	4.982	6.064	6.550

DEGREES OF FREEDOM 13

p \ ε	.995	.99	.95	.90	.75	.50	.25	.10	.05	.01	.005
.2500	-.015	.055	.242	.341	.511	.715	.943	1.182	1.345	1.707	1.862
.1500	.346	.411	.592	.693	.874	1.099	1.362	1.645	1.842	2.285	2.476
.1000	.568	.631	.817	.923	1.115	1.360	1.651	1.967	2.188	2.690	2.911
.0650	.766	.833	1.025	1.137	1.342	1.608	1.927	2.276	2.523	3.085	3.330
.0400	.962	1.030	1.232	1.351	1.572	1.860	2.210	2.595	2.868	3.492	3.766
.0250	1.128	1.200	1.412	1.537	1.773	2.083	2.461	2.879	3.176	3.854	4.158
.0100	1.411	1.489	1.721	1.861	2.124	2.474	2.902	3.380	3.720	4.500	4.850
.0040	1.657	1.738	1.992	2.145	2.435	2.821	3.297	3.829	4.207	5.079	5.462
.0025	1.769	1.857	2.120	2.279	2.582	2.986	3.485	4.042	4.441	5.356	5.762
.0010	1.978	2.071	2.353	2.524	2.851	3.288	3.829	4.435	4.867	5.864	6.313

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PERCENTAGE POINTS OF t, THE NON-CENTRAL t-STATISTIC. THE ENTRIES IN THE TABLE GIVE THE VALUES OF x SUCH THAT $Pr\ [t/\sqrt{f} > x] = \epsilon$

f is the number of degrees of freedom; the non-centrality parameter is $\sqrt{f+1}\,K_p$.
K_p is the standardized normal deviate exceeded with probability p.

DEGREES OF FREEDOM 14

p \ ε	.995	.99	.95	.90	.75	.50	.25	.10	.05	.01	.005
.2500	.009	.076	.256	.352	.516	.712	.931	1.158	1.312	1.652	1.796
.1500	.365	.430	.605	.703	.878	1.095	1.347	1.615	1.801	2.214	2.395
.1000	.587	.652	.830	.933	1.119	1.354	1.632	1.932	2.141	2.609	2.812
.0650	.788	.852	1.038	1.146	1.345	1.601	1.906	2.238	2.469	2.993	3.221
.0400	.983	1.049	1.245	1.360	1.574	1.852	2.186	2.552	2.808	3.389	3.642
.0250	1.150	1.219	1.425	1.547	1.776	2.074	2.435	2.831	3.110	3.742	4.016
.0100	1.432	1.508	1.736	1.871	2.127	2.463	2.873	3.325	3.644	4.372	4.690
.0040	1.678	1.762	2.008	2.156	2.437	2.809	3.263	3.766	4.122	4.934	5.290
.0025	1.795	1.879	2.137	2.291	2.584	2.973	3.449	3.977	4.350	5.206	5.574
.0010	2.005	2.094	2.370	2.537	2.853	3.274	3.790	4.363	4.769	5.700	6.100

DEGREES OF FREEDOM 15

p \ ε	.995	.99	.95	.90	.75	.50	.25	.10	.05	.01	.005
.2500	.031	.096	.269	.361	.520	.709	.920	1.137	1.283	1.604	1.737
.1500	.388	.447	.617	.712	.881	1.091	1.333	1.589	1.764	2.154	2.318
.1000	.606	.668	.842	.942	1.122	1.350	1.617	1.902	2.099	2.539	2.730
.0650	.807	.868	1.050	1.155	1.348	1.595	1.888	2.204	2.423	2.915	3.124
.0400	1.002	1.065	1.257	1.369	1.577	1.845	2.166	2.513	2.755	3.300	3.536
.0250	1.168	1.236	1.438	1.557	1.778	2.067	2.412	2.789	3.053	3.646	3.900
.0100	1.454	1.527	1.749	1.881	2.129	2.454	2.846	3.276	3.577	4.259	4.550
.0040	1.702	1.781	2.022	2.167	2.440	2.798	3.233	3.712	4.047	4.809	5.140
.0025	1.817	1.900	2.152	2.302	2.587	2.962	3.418	3.919	4.272	5.069	5.421
.0010	2.029	2.117	2.386	2.548	2.856	3.261	3.756	4.301	4.683	5.550	5.929

DEGREES OF FREEDOM 16

p \ ε	.995	.99	.95	.90	.75	.50	.25	.10	.05	.01	.005
.2500	.052	.113	.280	.370	.524	.707	.910	1.118	1.258	1.561	1.689
.1500	.405	.462	.628	.720	.884	1.087	1.321	1.566	1.733	2.100	2.258
.1000	.622	.682	.853	.950	1.125	1.345	1.602	1.876	2.063	2.479	2.655
.0650	.823	.883	1.061	1.164	1.351	1.590	1.872	2.173	2.382	2.844	3.044
.0400	1.017	1.081	1.269	1.378	1.580	1.839	2.147	2.480	2.710	3.224	3.443
.0250	1.186	1.253	1.450	1.566	1.781	2.060	2.392	2.753	3.003	3.563	3.800
.0100	1.474	1.545	1.762	1.891	2.132	2.446	2.823	3.233	3.520	4.162	4.434
.0040	1.720	1.800	2.036	2.177	2.442	2.789	3.207	3.664	3.982	4.700	5.010
.0025	1.837	1.920	2.165	2.312	2.589	2.952	3.390	3.869	4.204	4.955	5.280
.0010	2.050	2.137	2.401	2.559	2.858	3.250	3.726	4.245	4.609	5.430	5.778

DEGREES OF FREEDOM 17

p \ ε	.995	.99	.95	.90	.75	.50	.25	.10	.05	.01	.005
.2500	.069	.127	.291	.378	.527	.705	.901	1.102	1.235	1.524	1.643
.1500	.419	.476	.637	.728	.887	1.084	1.310	1.545	1.704	2.054	2.200
.1000	.637	.697	.863	.957	1.128	1.341	1.589	1.852	2.031	2.425	2.595
.0650	.837	.898	1.071	1.171	1.354	1.586	1.857	2.147	2.345	2.786	2.970
.0400	1.033	1.096	1.279	1.386	1.582	1.834	2.131	2.450	2.669	3.156	3.361
.0250	1.204	1.267	1.460	1.574	1.784	2.054	2.375	2.720	2.958	3.486	3.713
.0100	1.490	1.561	1.773	1.899	2.134	2.439	2.802	3.196	3.468	4.076	4.335
.0040	1.739	1.817	2.049	2.186	2.444	2.781	3.184	3.621	3.925	4.604	4.891
.0025	1.857	1.938	2.178	2.322	2.592	2.943	3.366	3.824	4.143	4.854	5.160
.0010	2.069	2.157	2.415	2.570	2.860	3.241	3.699	4.196	4.543	5.317	5.650

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PERCENTAGE POINTS OF t, THE NON-CENTRAL t-STATISTIC. THE ENTRIES IN THE TABLE GIVE THE VALUES OF x SUCH THAT $Pr\ [t/\sqrt{f} > x] = \epsilon$

f is the number of degrees of freedom; the non-centrality parameter is $\sqrt{f+1}\,K_p$.
K_p is the standardized normal deviate exceeded with probability p.

DEGREES OF FREEDOM 18

p \ ε	.995	.99	.95	.90	.75	.50	.25	.10	.05	.01	.005
.2500	.083	.144	.301	.385	.531	.703	.894	1.087	1.215	1.490	1.603
.1500	.431	.488	.647	.734	.890	1.082	1.300	1.526	1.679	2.012	2.150
.1000	.655	.710	.872	.964	1.130	1.338	1.578	1.830	2.002	2.378	2.536
.0650	.853	.911	1.080	1.178	1.356	1.582	1.844	2.123	2.313	2.732	2.909
.0400	1.050	1.109	1.289	1.393	1.585	1.829	2.117	2.423	2.633	3.097	3.294
.0250	1.219	1.281	1.470	1.581	1.786	2.049	2.359	2.690	2.918	3.420	3.637
.0100	1.506	1.576	1.784	1.907	2.136	2.432	2.784	3.162	3.422	4.000	4.243
.0040	1.758	1.832	2.060	2.195	2.447	2.774	3.163	3.583	3.873	4.516	4.793
.0025	1.875	1.954	2.190	2.331	2.594	2.936	3.344	3.784	4.089	4.766	5.050
.0010	2.088	2.174	2.428	2.579	2.863	3.232	3.675	4.152	4.484	5.217	5.533

DEGREES OF FREEDOM 19

p \ ε	.995	.99	.95	.90	.75	.50	.25	.10	.05	.01	.005
.2500	.102	.156	.310	.392	.534	.702	.887	1.073	1.197	1.461	1.570
.1500	.448	.502	.656	.741	.893	1.079	1.291	1.509	1.657	1.975	2.107
.1000	.665	.721	.880	.971	1.133	1.335	1.568	1.811	1.976	2.334	2.487
.0650	.866	.923	1.089	1.185	1.359	1.578	1.833	2.101	2.284	2.682	2.850
.0400	1.063	1.122	1.298	1.400	1.587	1.825	2.104	2.398	2.600	3.042	3.229
.0250	1.231	1.295	1.480	1.588	1.788	2.044	2.344	2.664	2.882	3.364	3.568
.0100	1.522	1.590	1.795	1.915	2.139	2.427	2.767	3.131	3.381	3.932	4.163
.0040	1.774	1.849	2.071	2.203	2.449	2.767	3.144	3.549	3.826	4.442	4.700
.0025	1.891	1.970	2.202	2.340	2.597	2.929	3.324	3.748	4.040	4.684	4.958
.0010	2.107	2.190	2.440	2.588	2.865	3.225	3.653	4.113	4.431	5.133	5.434

DEGREES OF FREEDOM 20

p \ ε	.995	.99	.95	.90	.75	.50	.25	.10	.05	.01	.005
.2500	.114	.167	.318	.398	.537	.700	.880	1.061	1.180	1.432	1.537
.1500	.460	.513	.663	.747	.896	1.077	1.282	1.494	1.636	1.940	2.068
.1000	.677	.732	.888	.977	1.135	1.332	1.558	1.793	1.952	2.296	2.442
.0650	.875	.935	1.098	1.191	1.361	1.575	1.822	2.081	2.257	2.640	2.800
.0400	1.074	1.134	1.307	1.407	1.589	1.821	2.092	2.377	2.571	2.995	3.171
.0250	1.244	1.307	1.489	1.595	1.791	2.040	2.331	2.639	2.850	3.311	3.505
.0100	1.536	1.604	1.804	1.923	2.141	2.422	2.752	3.103	3.343	3.869	4.093
.0040	1.789	1.863	2.081	2.211	2.452	2.761	3.127	3.517	3.785	4.373	4.621
.0025	1.909	1.984	2.212	2.348	2.599	2.923	3.306	3.715	3.996	4.611	4.875
.0010	2.123	2.206	2.451	2.597	2.868	3.218	3.633	4.077	4.382	5.054	5.334

DEGREES OF FREEDOM 21

p \ ε	.995	.99	.95	.90	.75	.50	.25	.10	.05	.01	.005
.2500	.125	.178	.325	.404	.539	.699	.874	1.049	1.165	1.408	1.508
.1500	.471	.523	.671	.753	.898	1.075	1.275	1.480	1.617	1.911	2.031
.1000	.691	.742	.896	.983	1.138	1.330	1.550	1.777	1.931	2.260	2.396
.0650	.888	.945	1.105	1.197	1.363	1.572	1.812	2.063	2.233	2.600	2.754
.0400	1.086	1.145	1.314	1.413	1.592	1.818	2.081	2.356	2.544	2.950	3.119
.0250	1.258	1.318	1.497	1.602	1.793	2.036	2.319	2.617	2.821	3.263	3.445
.0100	1.550	1.616	1.813	1.929	2.143	2.417	2.738	3.078	3.309	3.815	4.027
.0040	1.804	1.875	2.091	2.218	2.454	2.756	3.111	3.489	3.747	4.310	4.550
.0025	1.924	1.999	2.223	2.355	2.601	2.917	3.290	3.685	3.956	4.546	4.793
.0010	2.140	2.221	2.462	2.605	2.870	3.212	3.615	4.045	4.338	4.980	5.250

PERCENTAGE POINTS OF t, THE NON-CENTRAL t-STATISTIC. THE ENTRIES IN THE TABLE GIVE THE VALUES OF x SUCH THAT $Pr\ [t/\sqrt{f} > x] = \epsilon$

f is the number of degrees of freedom; the non-centrality parameter is $\sqrt{f+1}\ K_p$.
K_p is the standardized normal deviate exceeded with probability p.

DEGREES OF FREEDOM 22

p \ ε	.995	.99	.95	.90	.75	.50	.25	.10	.05	.01	.005
.2500	.133	.190	.332	.410	.542	.698	.868	1.039	1.151	1.385	1.483
.1500	.480	.530	.677	.758	.900	1.073	1.268	1.467	1.600	1.882	1.996
.1000	.703	.754	.903	.988	1.140	1.328	1.542	1.763	1.911	2.228	2.362
.0650	.903	.955	1.112	1.203	1.366	1.569	1.803	2.047	2.211	2.564	2.710
.0400	1.100	1.156	1.322	1.418	1.594	1.815	2.070	2.338	2.519	2.909	3.071
.0250	1.269	1.329	1.505	1.608	1.795	2.032	2.308	2.597	2.793	3.217	[illegible]
.0100	1.563	1.627	1.822	1.936	2.146	2.413	2.725	3.054	3.278	3.763	3.968
.0040	1.816	1.889	2.101	2.225	2.456	2.751	3.097	3.463	3.712	4.253	4.481
.0025	1.936	2.012	2.232	2.363	2.603	2.912	3.274	3.658	3.919	4.486	4.729
.0010	2.157	2.235	2.472	2.613	2.873	3.206	3.599	4.015	4.298	4.914	5.178

DEGREES OF FREEDOM 23

p \ ε	.995	.99	.95	.90	.75	.50	.25	.10	.05	.01	.005
.2500	.150	.201	.339	.415	.544	.697	.863	1.029	1.138	1.365	1.457
.1500	.489	.543	.684	.763	.902	1.072	1.261	1.455	1.584	1.856	1.968
.1000	.712	.763	.910	.993	1.142	1.326	1.534	1.749	1.893	2.200	2.327
.0650	.911	.965	1.119	1.208	1.368	1.567	1.795	2.032	2.190	2.531	2.674
.0400	1.108	1.165	1.329	1.424	1.596	1.812	2.061	2.321	2.496	2.873	3.032
.0250	1.280	1.338	1.513	1.613	1.797	2.029	2.298	2.579	2.769	3.177	3.350
.0100	1.575	1.638	1.830	1.942	2.148	2.409	2.713	3.033	3.249	3.716	3.912
.0040	1.829	1.901	2.109	2.232	2.458	2.747	3.084	3.439	3.680	4.203	4.417
.0025	1.952	2.024	2.241	2.369	2.606	2.908	3.260	3.633	3.885	4.432	4.664
.0010	2.171	2.249	2.482	2.620	2.875	3.201	3.584	3.988	4.261	4.857	5.107

DEGREES OF FREEDOM 24

p \ ε	.995	.99	.95	.90	.75	.50	.25	.10	.05	.01	.005
.2500	.158	.209	.346	.420	.547	.696	.859	1.020	1.126	1.346	1.437
.1500	.503	.553	.690	.768	.905	1.070	1.256	1.444	1.569	1.833	1.941
.1000	.716	.770	.916	.998	1.144	1.324	1.527	1.737	1.876	2.173	2.293
.0650	.920	.972	1.126	1.213	1.370	1.565	1.787	2.017	2.171	2.500	2.638
.0400	1.117	1.174	1.336	1.429	1.598	1.809	2.053	2.305	2.475	2.838	2.991
.0250	1.291	1.350	1.520	1.619	1.799	2.026	2.288	2.562	2.746	3.140	3.305
.0100	1.586	1.650	1.838	1.948	2.150	2.406	2.702	3.013	3.223	3.675	3.866
.0040	1.842	1.912	2.117	2.238	2.460	2.743	3.072	3.417	3.650	4.153	4.361
.0025	1.964	2.036	2.250	2.376	2.608	2.903	3.248	3.609	3.854	4.384	4.606
.0010	2.183	2.261	2.491	2.627	2.877	3.197	3.569	3.962	4.228	4.803	5.044

DEGREES OF FREEDOM 29

p \ ε	.995	.99	.95	.90	.75	.50	.25	.10	.05	.01	.005
.2500	.203	.249	.372	.440	.556	.692	.839	.983	1.077	1.270	1.346
.1500	.540	.585	.717	.788	.913	1.064	1.231	1.399	1.509	1.737	1.831
.1000	.759	.809	.943	1.019	1.153	1.317	1.500	1.685	1.808	2.064	2.167
.0650	.960	1.012	1.154	1.234	1.378	1.556	1.756	1.960	2.095	2.380	2.497
.0400	1.164	1.214	1.365	1.451	1.607	1.799	2.018	2.242	2.390	2.704	2.832
.0250	1.336	1.390	1.550	1.642	1.808	2.015	2.250	2.492	2.653	2.994	3.133
.0100	1.634	1.695	1.871	1.973	2.159	2.392	2.658	2.933	3.116	3.505	3.665
.0040	1.896	1.961	2.154	2.266	2.470	2.727	3.022	3.327	3.531	3.965	4.139
.0025	2.018	2.087	2.287	2.404	2.618	2.887	3.195	3.515	3.729	4.183	4.372
.0010	2.241	2.315	2.531	2.657	2.888	3.178	3.513	3.859	4.091	4.585	4.788

PERCENTAGE POINTS OF t, THE NON-CENTRAL t-STATISTIC. THE ENTRIES IN THE TABLE GIVE THE VALUES OF x SUCH THAT $Pr\ [t/\sqrt{f} > x] = \epsilon$

f is the number of degrees of freedom; the non-centrality parameter is $\sqrt{f+1}\, K_p$.
K_p is the standardized normal deviate exceeded with probability p.

DEGREES OF FREEDOM 34

p \ ε	.995	.99	.95	.90	.75	.50	.25	.10	.05	.01	.005
.2500	.230	.269	.393	.457	.564	.690	.824	.956	1.040	1.212	1.282
.1500	.568	.612	.737	.804	.921	1.060	1.213	1.368	1.464	1.669	1.750
.1000	.789	.835	.965	1.035	1.160	1.311	1.479	1.647	1.757	1.986	2.077
.0650	.997	1.041	1.176	1.251	1.386	1.550	1.733	1.918	2.039	2.293	2.394
.0400	1.200	1.248	1.389	1.469	1.614	1.792	1.992	2.195	2.328	2.607	2.718
.0250	1.372	1.424	1.575	1.661	1.816	2.007	2.222	2.441	2.585	2.887	3.010
.0100	1.673	1.730	1.898	1.994	2.167	2.382	2.626	2.874	3.038	3.383	3.524
.0040	1.939	2.002	2.183	2.288	2.479	2.716	2.986	3.261	3.443	3.847	3.982
.0025	2.064	2.128	2.318	2.427	2.627	2.875	3.157	3.446	3.637	4.040	4.205
.0010	2.289	2.360	2.564	2.682	2.897	3.165	3.471	3.784	3.991	4.428	4.605

DEGREES OF FREEDOM 39

p \ ε	.995	.99	.95	.90	.75	.50	.25	.10	.05	.01	.005
.2500	.265	.303	.410	.469	.570	.688	.813	.935	1.012	1.169	1.232
.1500	.604	.645	.755	.817	.927	1.057	1.199	1.340	1.430	1.617	1.689
.1000	.824	.865	.983	1.049	1.166	1.308	1.463	1.618	1.719	1.926	2.009
.0650	1.027	1.067	1.194	1.265	1.392	1.545	1.715	1.885	1.996	2.225	2.318
.0400	1.229	1.275	1.408	1.484	1.620	1.787	1.972	2.158	2.280	2.533	2.632
.0250	1.405	1.453	1.595	1.676	1.822	2.001	2.200	2.401	2.533	2.806	2.915
.0100	1.708	1.762	1.920	2.011	2.174	2.375	2.601	2.829	2.978	3.291	3.414
.0040	1.974	2.034	2.207	2.306	2.486	2.708	2.958	3.210	3.376	3.724	3.861
.0025	2.102	2.162	2.343	2.447	2.634	2.866	3.127	3.393	3.567	3.930	4.078
.0010	2.330	2.397	2.591	2.702	2.905	3.155	3.438	3.726	3.915	4.309	4.466

DEGREES OF FREEDOM 44

p \ ε	.995	.99	.95	.90	.75	.50	.25	.10	.05	.01	.005
.2500	.287	.324	.425	.481	.576	.686	.804	.918	.989	1.135	1.191
.1500	.626	.664	.770	.828	.932	1.055	1.188	1.319	1.403	1.574	1.640
.1000	.846	.885	.997	1.060	1.171	1.305	1.451	1.595	1.688	1.878	1.954
.0650	1.050	1.092	1.210	1.277	1.397	1.542	1.701	1.859	1.961	2.172	2.255
.0400	1.253	1.298	1.424	1.497	1.626	1.783	1.956	2.129	2.242	2.474	2.565
.0250	1.431	1.477	1.612	1.690	1.828	1.996	2.183	2.370	2.491	2.743	2.841
.0100	1.733	1.786	1.939	2.025	2.180	2.369	2.581	2.793	2.930	3.217	3.330
.0040	2.006	2.062	2.228	2.322	2.492	2.701	2.935	3.170	3.323	3.640	3.768
.0025	2.133	2.192	2.364	2.463	2.641	2.859	3.104	3.350	3.511	3.844	3.976
.0010	2.364	2.428	2.613	2.720	2.912	3.148	3.413	3.679	3.854	4.215	4.361

DEGREES OF FREEDOM 49

p \ ε	.995	.99	.95	.90	.75	.50	.25	.10	.05	.01	.005
.2500	.306	.341	.438	.490	.580	.685	.796	.903	.971	1.107	1.160
.1500	.645	.681	.782	.837	.936	1.053	1.179	1.301	1.380	1.539	1.602
.1000	.865	.902	1.009	1.070	1.176	1.302	1.440	1.576	1.663	1.839	1.909
.0650	1.071	1.110	1.224	1.287	1.402	1.539	1.689	1.837	1.933	2.128	2.206
.0400	1.275	1.317	1.438	1.508	1.631	1.779	1.943	2.105	2.210	2.426	2.510
.0250	1.453	1.498	1.627	1.701	1.833	1.992	2.168	2.344	2.457	2.690	2.783
.0100	1.762	1.810	1.956	2.038	2.185	2.365	2.564	2.763	2.892	3.156	3.262
.0040	2.033	2.085	2.245	2.336	2.498	2.696	2.917	3.137	3.280	3.574	3.690
.0025	2.160	2.217	2.383	2.477	2.647	2.854	3.085	3.315	3.465	3.773	3.896
.0010	2.394	2.456	2.633	2.735	2.918	3.142	3.392	3.641	3.804	4.138	4.271

APPENDIX D

LIST OF ORGANIZATIONS CONSULTED

D.1 <u>Military</u>:

1. Ammunition Engineering Branch
   Industrial Engineering Division
   Picatinny Arsenal
   Dover, New Jersey

2. Airborne Systems Test Activity
   Army Field Evaluation Agency
   Quartermaster Research and Engineering Command
   Yuma, Arizona

3. Airborne Service Test Division
   U. S. Army Airborne and Electronics Board
   Fort Bragg, North Carolina

4. 6511th Test Group (Parachute), USAF
   Naval Auxiliary Air Station
   El Centro, California

5. Functional Textiles Section
   Textile Branch
   Materials Laboratory
   Wright Air Development Division
   Wright-Patterson Air Force Base, Ohio

6. Technical Services Branch
   Clothing and Survival Equipment Division
   Middletown Air Materiel Area
   Olmsted Air Force Base, Pennsylvania

7. Textile Engineering Laboratory Branch
   Textile, Clothing and Footwear Division
   Quartermaster Research and Engineering Command
   Natick, Massachusetts

D.2 Industrial:

1. Pioneer Parachute Company, Inc.
   Manchester, Connecticut

2. Buser Silk Corporation
   Paterson, New Jersey

3. Phoenix Trimming Company
   Chicago, Illinois

4. Narricot Corporation
   Philadelphia, Pennsylvania

5. Bally Ribbon Mills, Inc.
   Bally, Pennsylvania

6. Catalyst Research Corporation
   Baltimore, Maryland

7. Hercules Powder Company
   Wilmington, Delaware

8. Essex Mills, Inc.
   Pawtucket, Rhode Island

9. Fox Specialty Co., Inc.
   Lowell, Massachusetts

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